Monday, 1/30/17 **1.2 Graphs of Functions**

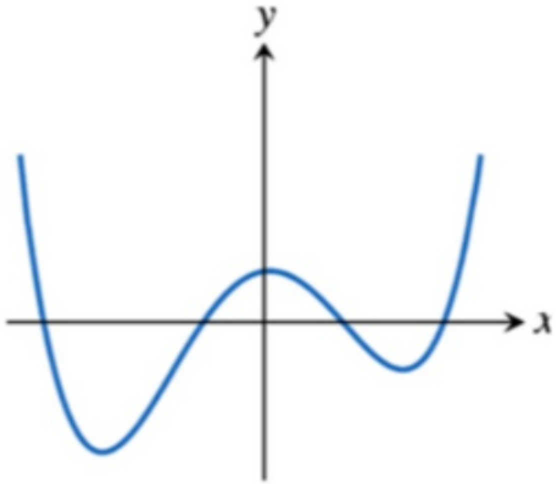
1. Determine intervals on which a function is increasin decreasing or constant.
2. Identify relative maximum and minimum values of functions.
3. Identify even vs. odd functions algebraically & graphically.

Concept: Continuity

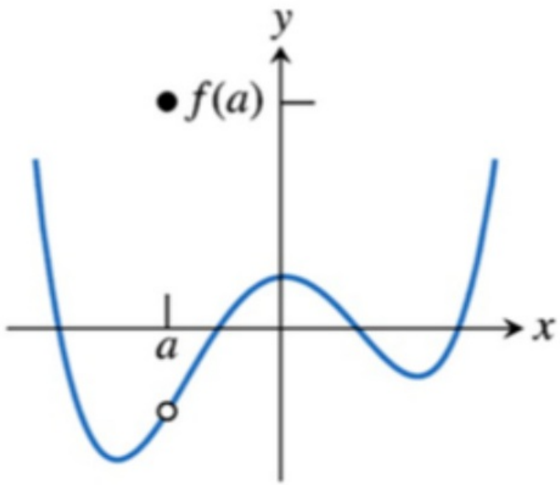
- A graph is continuous if it can be sketched without lifting your pencil.
- Graphs become discontinuous 3 ways:
  1. Holes (removable discontinuity)
  2. Jumps (jump discontinuity)
  3. Vertical asymptotes (infinite discontinuity)

- Continuity can relate to the whole function, an interval of values or a particular point.

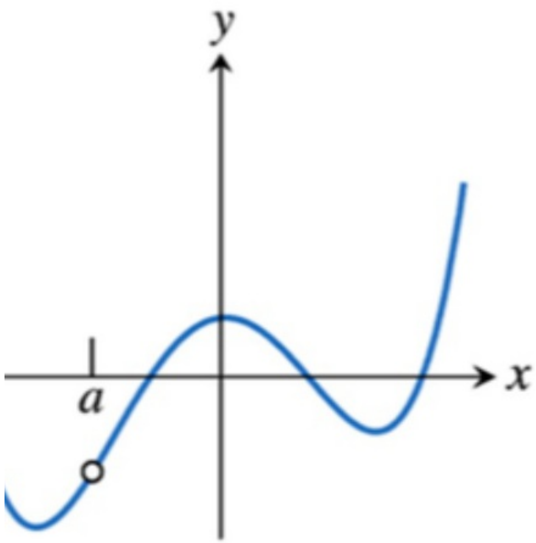
Continuity



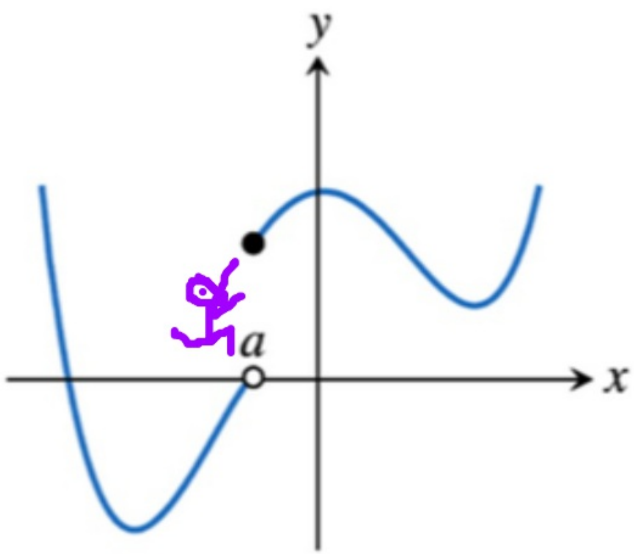
Continuous at all x



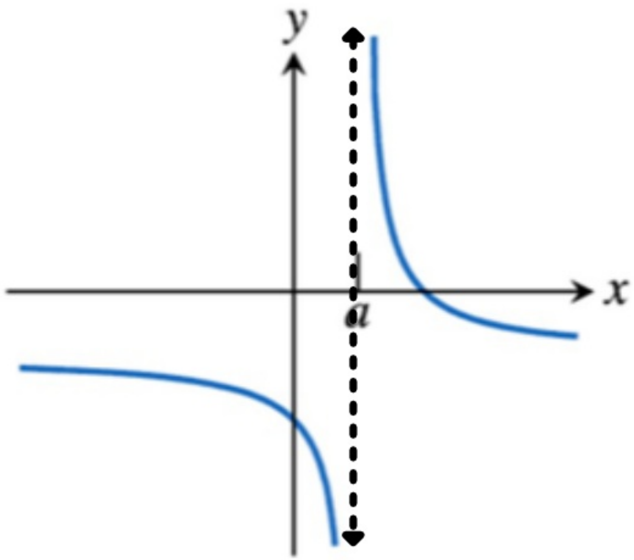
Removable discontinuity



emovable discontinuity



Jump discontinuity



Infinite discontinuity

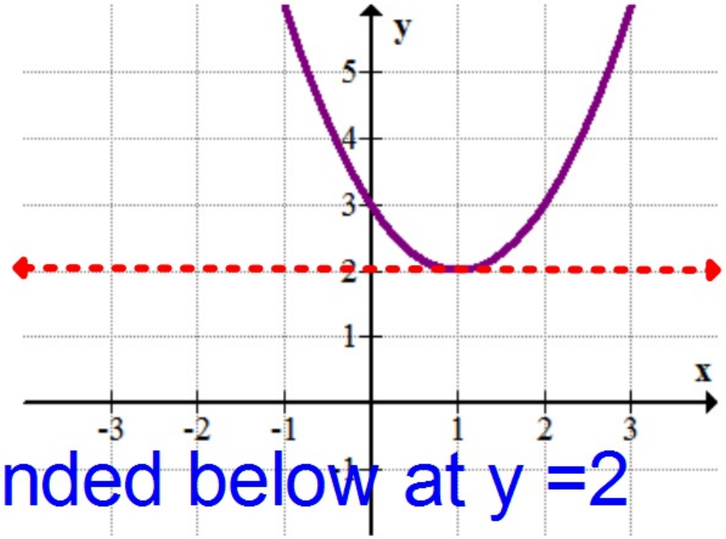
Concept: Boundedness

- Bounded below - graph has an absolute minimum, can draw a horizontal line below the entire graph

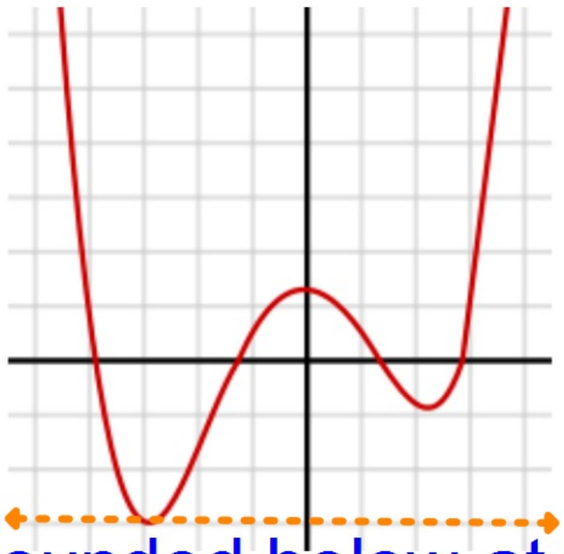
- Bounded above - graph has an absolute maximum, can draw a horizontal line above the entire graph

- Unbounded - graph has neither an absolute maximum or absolute minimum (extends off to infinity at both ends)

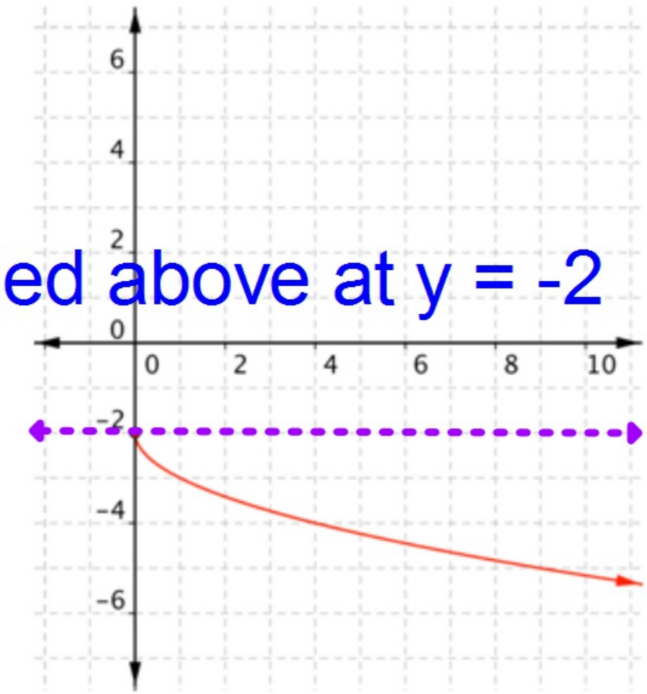
- Bounded - graph that has both an absolute maximum(s) and an absolute minimum(s) (can draw a horizontal line above & below the entire graph)



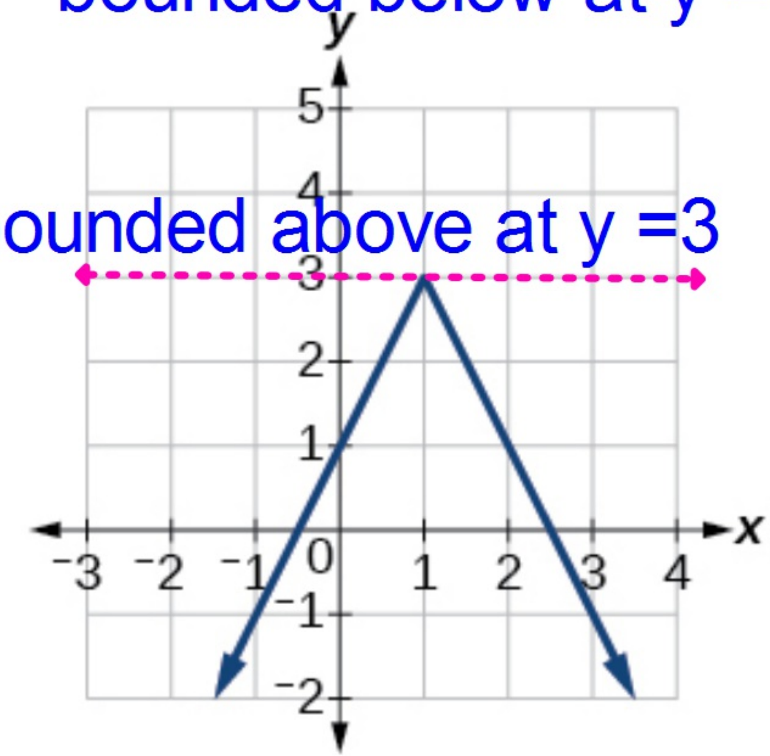
unded below at y =2



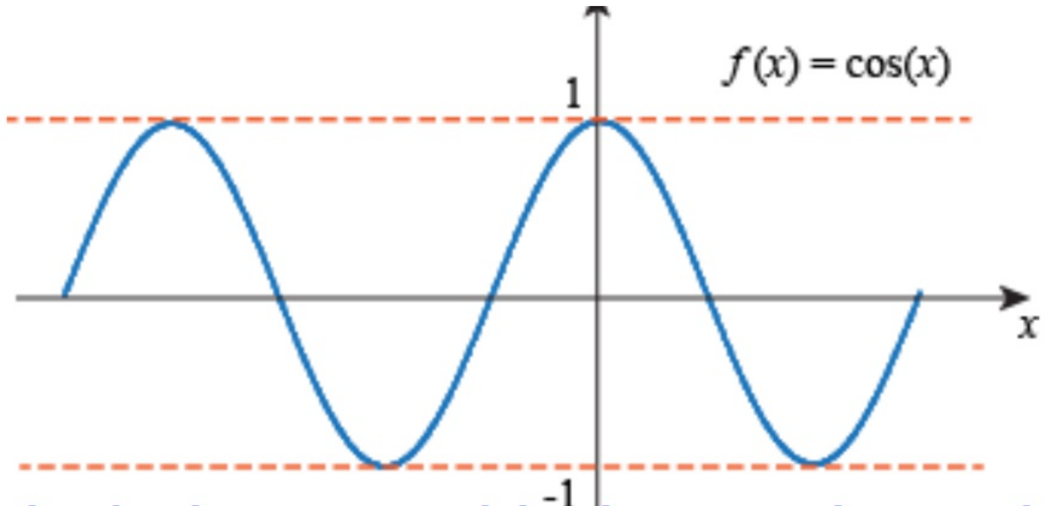
bounded below at y = -3



nded above at y = -2

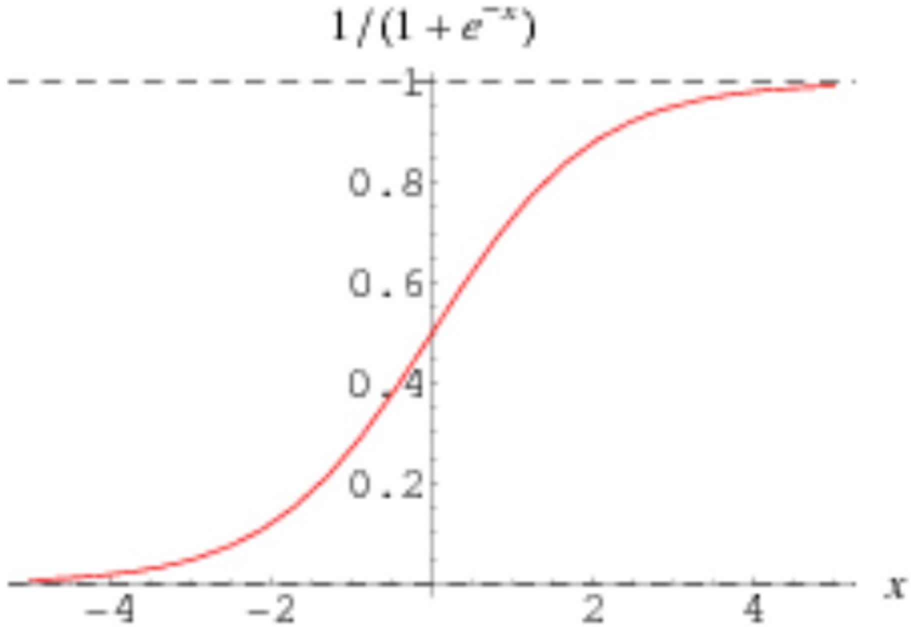


bounded above at y =3



bounded above and below so bounded

unbounded, no abs max or min



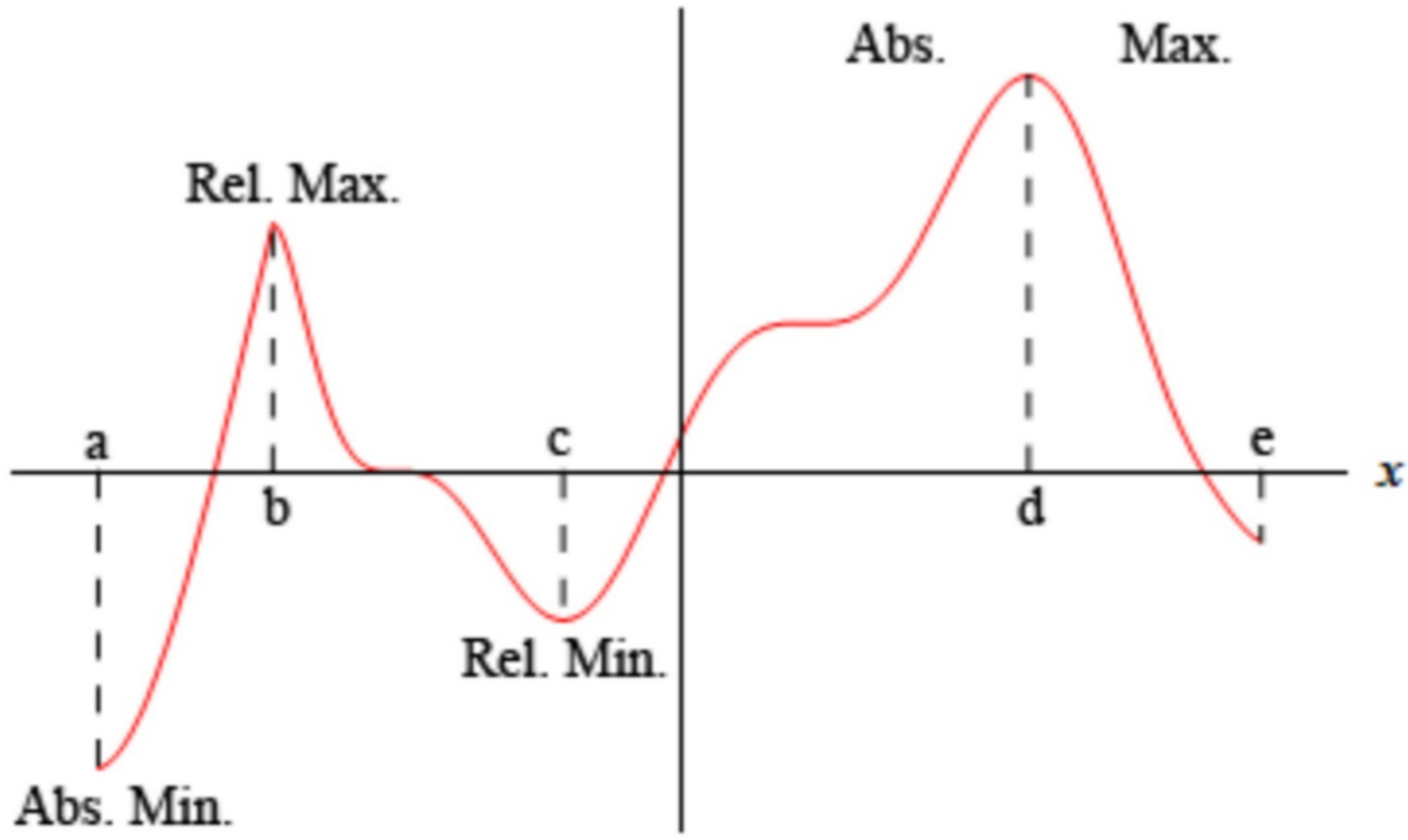
Concept: Relative & Absolute Extrema

- If a graph changes from increasing to descreasing or vice versa, then it has "peaks" and "valleys"

- The point (x,y) at the top of the peak = relative maximum

- The point (x,y) at the bottom of the valley = relative minimum

- If the point is the maximum or minimum value of the ENTIRE function, it is called an absolute maximum or absolute minimum

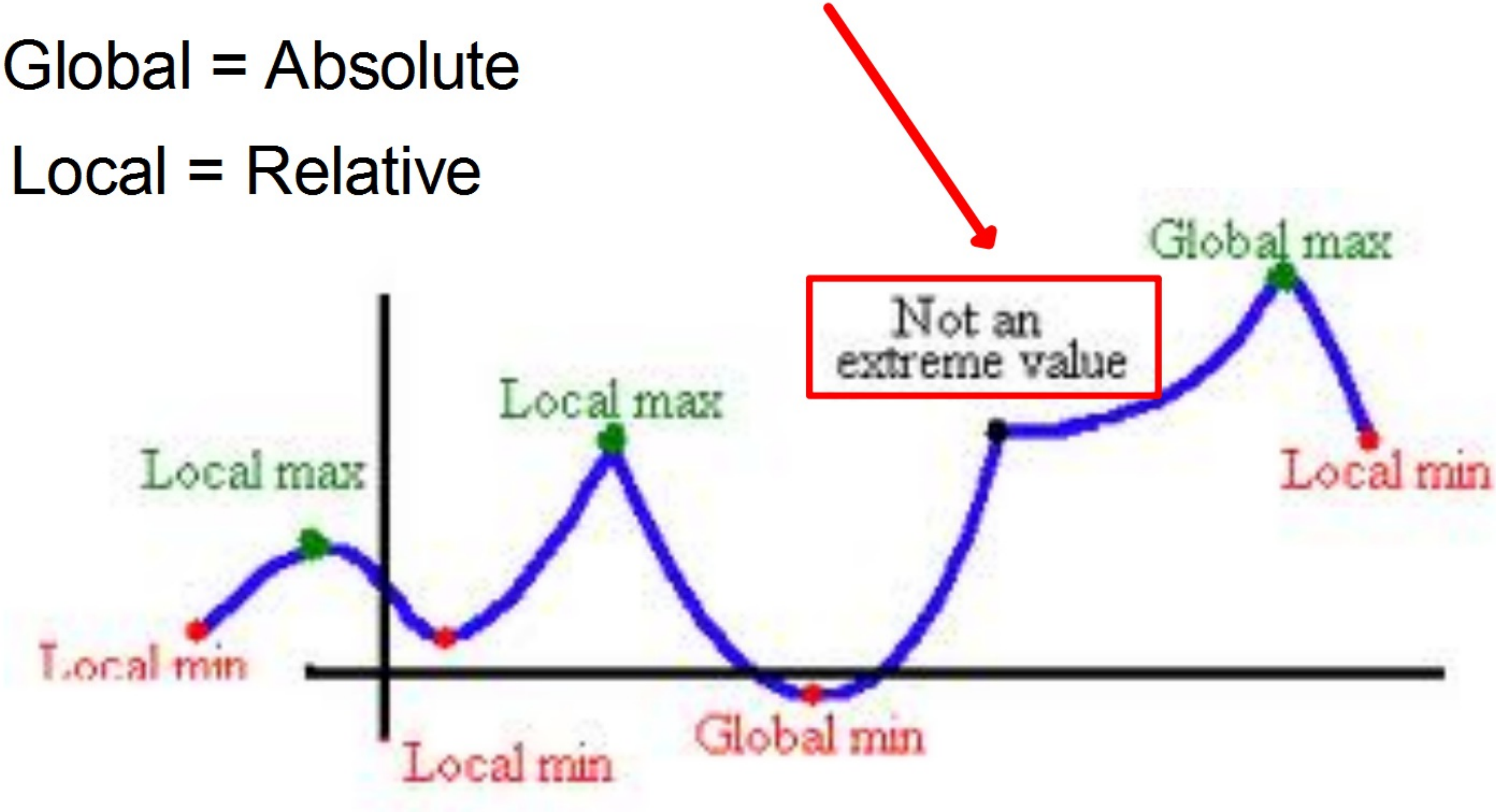


Abs.
Max.
Rel. Max.
a
b
c
d
e
x
Rel. Min.
Abs. Min.

WHY?

Global = Absolute

Local = Relative



Concept: Increasing & Decreasing Intervals

- At a particular point, a function is either increasing, decreasing or constant.
- It is easiest to figure out which is happening by looking at the graph.
- As you move from left to right:
  1. If graph has a positive slope = increasing
  2. If graph has a negative slope = decreasing
  3. If slope = 0, then it is constant
- Use interval notation to write (parenthesis) x-values

Ex 1: Use a graphing utility to approximate the relative maximum(s) and/or minimum(s) of each function.

a. $f(x) = 3x^2 - 4x - 2$

x

min at (.667, -3.33

b. $h(x) = -x^3 + x$

min at (-.577, -.385)

max at (.577, .385)

Increasing & Decreasing Intervals

Ex 2: Determine the open intervals on which each function is increasing or decreasing.

a. $f(x) = 3x^2 - 4x - 2$

Inc: $(.667, \infty)$

Dec: $(-\infty, .667)$

b. $h(x) = -x^3 + x$

Inc: $(-.577, .577)$

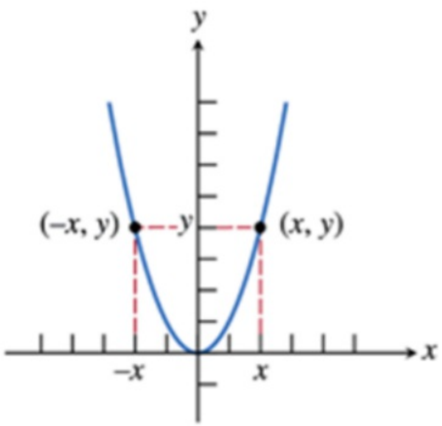
Dec: $(-\infty, -.577)$ $(.577, \infty)$

Concept: Symmetry

- y-axis symmetry - graph looks identical on the left & right side of the y-axis
  - Called an even function
  - Algebraic test: f(-x) = f(x)

Example: $f(x) = x^2$

Graphically



Numerically

x	$f(x)$
−3	9
−2	4
−1	1
1	1
2	4
3	9

Algebraically

For all x in the domain of f,

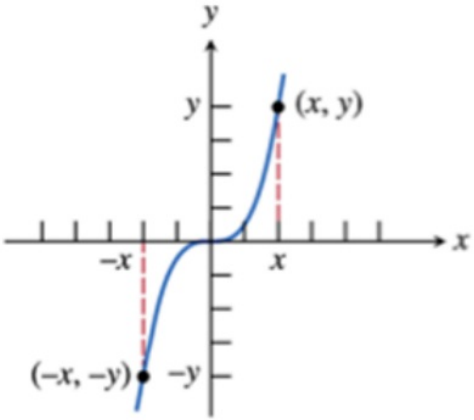
$$f(-x) = f(x)$$

Functions with this property (for example, x^n, n even) are **even** functions.

- Origin symmetry - graph can be rotated 180° and still looks the same
  - Called an odd function
  - Algebraic test: f(-x) = -f(x)

Example: $f(x) = x^3$

Graphically



Numerically

x	y
−3	−27
−2	−8
−1	−1
1	1
2	8
3	27

Algebraically

For all x in the domain of f,

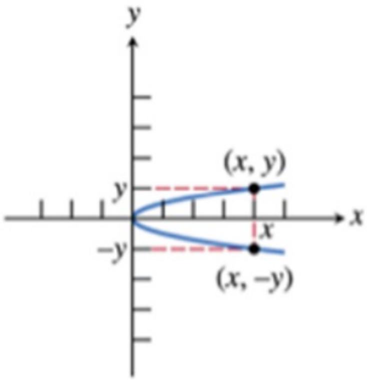
$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

- x-axis symmetry - graph looks identical above & below the x-axis
  - NOT a function
  - Algebraic test: plug -y into the equation & it will simplify back to original equation

Example: $x = y^2$

Graphically



Numerically

x	y
9	−3
4	−2
1	−1
1	1
4	2
9	3

Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Ex 3: Determine algebraically whether each function is even, odd or neither. Use a graphical solution to confirm your answer.

a. $h(x) = x^3 - 1$

b. $g(x) = \dfrac{x^3}{4 - x^2}$

c. $f(x) = x^2 + 1$

a. $h(-x) = (-x)^3 - 1$

$= -x^3 - 1$

neither

1.2 Ticket Out The Door

Vertex:________________________

Is this a max or a min? _________

Domain:________________________

Range: ________________________

Root(s): _______________________

Y-Intercept(s): _________________

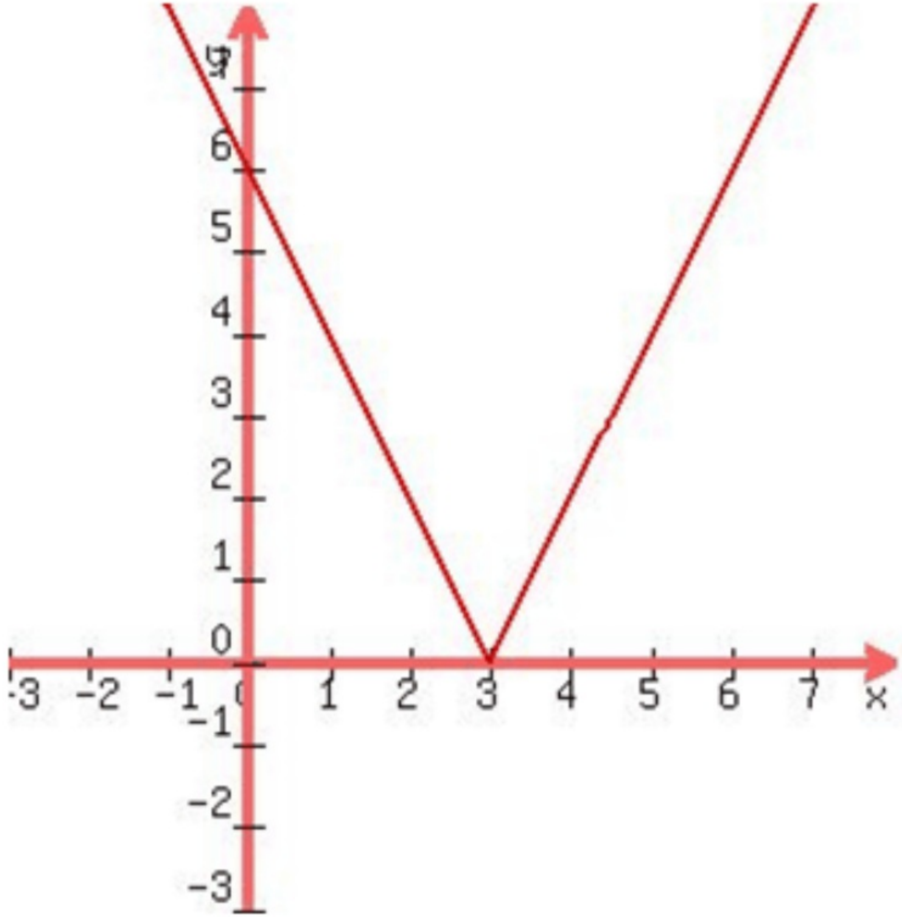
Interval increasing: ___________

Interval decreasing: ____________

End behavior: __________________

Axis of Symmetry:_____________

Even, Odd or Neither? _________



Think Tank 1.2

Hint Zone