Tuesday 2/7/17

1.6 Composition of Functions:

1. Evaluate combinations of functions.
2. Find compositions of one function with another function.

Why?

Most of the functions that you will encounter in calculus and in real life can be created by combining or modifyin other functions. Combining functions can sometimes help you better understand the big picture of real world situation, like in the number of bacteria in left over Chinese food.

Function Combinations

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the *sum*, *difference*, *product*, and *quotient* of f and g are defined as follows.

1. *Sum:* $(f + g)(x) = f(x) + g(x)$
2. *Difference:* $(f - g)(x) = f(x) - g(x)$
3. *Product:* $(fg)(x) = f(x) \cdot g(x)$
4. *Quotient:* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

In each case, the domain of the new function consists of all numbers that belong to both the domain of f and the domain of g.

Ex 1: Let $f(x) = x^3$ and $g(x) = \sqrt{x+1}$.

Find formulas of the functions and identify the domain of each new function.

(a) $f + g$

(b) $f - g$

(c) fg

(d) f / g

(a) $f(x) + g(x) = x^3 + \sqrt{x+1}$ with domain $[-1, \infty)$

(b) $f(x) - g(x) = x^3 - \sqrt{x+1}$ with domain $[-1, \infty)$

(c) $f(x)g(x) = x^3\sqrt{x+1}$ with domain $[-1, \infty)$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{x^3}{\sqrt{x+1}}$ with domain $(-1, \infty)$

Ex 2:

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$

(a) find $(f - g)(x)$. Then evaluate the difference when $x = 2$.

Ex 2: Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$

(b) find $(f \cdot g)(x)$.

Ex 3: Find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{g}{f}\right)(x)$ for the functions given by

$$f(x) = \sqrt{x} \qquad \text{and} \qquad g(x) = \sqrt{4 - x^2}.$$

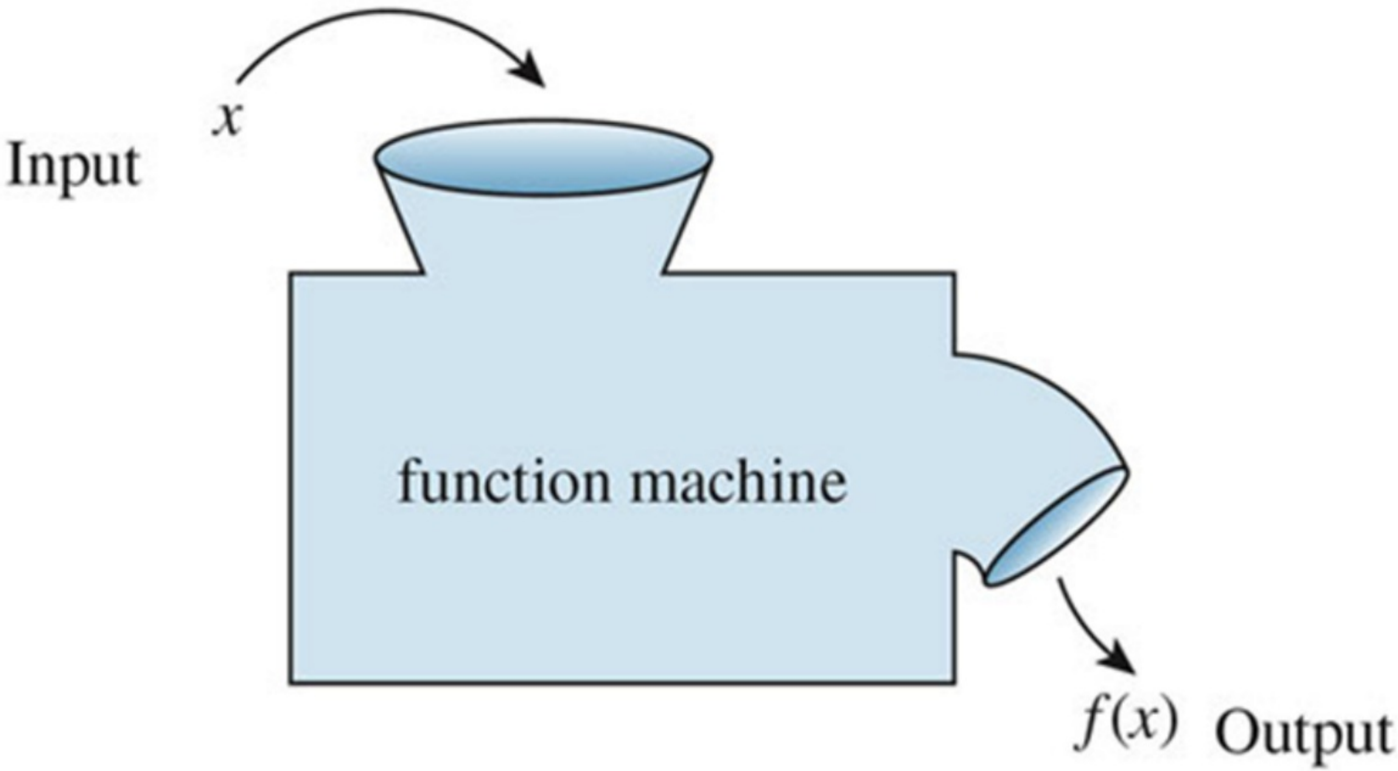
Then find the domains of f/g and g/f.

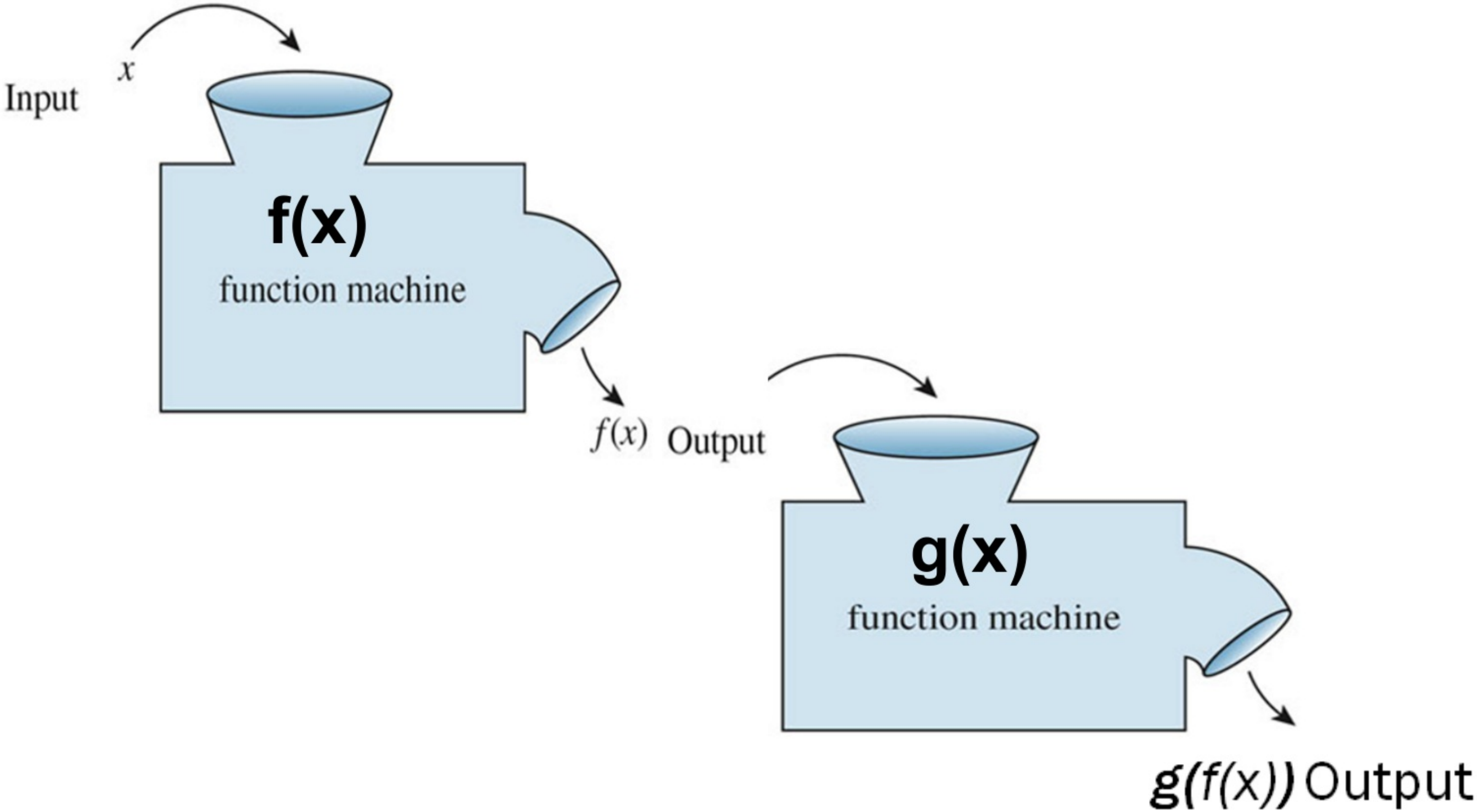
Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

f(x) = y




Input
x
f(x)
function machine
f(x) Output
g(x)
function machine
g(f(x)) Output

Ex 4: Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, find the following.

a. $(f \circ g)(x)$ **b.** $(g \circ f)(x)$ **c.** $(g \circ f)(-2)$

Ex 5:

Given $f(x) = x^2 - 9$ and $g(x) = \sqrt{9 - x^2}$, find the composition $(f \circ g)(x)$. Then find the domain of $(f \circ g)$.

Decomposing Functions

Ex 6: Find f and g such that $h(x) = f(g(x))$.

$$h(x) = \sqrt{x^2 + 5}$$

One possible decomposition:

Another possibility:

Ex 6: Write the function $h(x) = (3x-5)^3$ as a composition of two functions.

Ex 7: Write the function $h(x) = \frac{1}{(x-2)^2}$ as a composition of two functions.