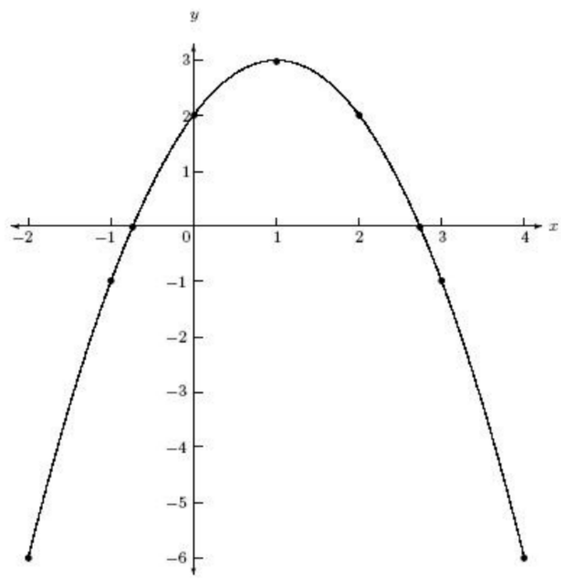
Chapter 2: Differentiation

2.1 Tangent Line & Differentiability

What is a tangent line?
a **line** which intersects a differentiable curve at a point where the slope of the curve equals the slope of the **line**



What is a secant line?
average rate of change between two points

The Limit Definition of the Derivative of a Function

If $f(x)$ is a differentiable function, its derivative function, $f'(x)$, is defined to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

NOTE: There are other notations for $f'(x)$. We will also use y' and $\frac{dy}{dx}$

The derivative can be used to find the slope of the tangent line to a graph at that point. A function that has a derivative at a point is said to be differentiable at that point.

Given $f(x)=\sqrt{x-2}$

(a) Graph $f(x)$. Describe what the *y*-values are doing and ***how*** they are doing it.

(b) Find $f'(x)$ using the limit definition of the derivative.

(b) Find the slope of $f(x)$ at (i) $x=6$ (ii) $x=16$ (iii) $x=27$ (iii) $x=2$.

(c) Describe the values of *x* for which $f(x)$ is (i) continuous (ii) differentiable.

(d) Write the equation of the tangent line, in Taylor form, to $f(x)$ at $x=27$.

Modified form of the limit definition of the derivative

The **numeric value** of the derivative of a function $f(x)$ at a point $(c, f(c))$ is given as

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

For $f(x) = \frac{x}{x+1}$,

(a) Find $f'(1)$ directly using the modified definition of the derivative.

(b) Find the equation of the <u>normal</u> line to $f(x)$ at $x = 1$.

Using 2 points, $(a, f(a))$ and $(b, f(b))$	Using 1 point $(c, f(c))$
The following are equivalent: • Slope of the secant line • $\dfrac{f(b)-f(a)}{b-a}$ • Average rate of change on the interval $[a,b]$	**The following are equivalent:** • Slope of the tangent line • $\lim_{h\to 0}\dfrac{f(c+h)-f(c)}{h}$ • $f'(c)$ • Derivative at $x=c$ • Instantaneous rate of change at $x=c$

Differentiability at a point $x = c$.

A function $f(x)$ is differentiable at $x = c$ if and only if $\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = L = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$, where L is a finite value.

*This basically says that to be differentiable at a point $(c, f(c))$, the graph must be continuous (connected) first, but must connect in a way that the slopes merge into each other smoothly.

SMOOOOOTHLY CONNECTED

To be differentiable, a function must be continuous and smooth.

Derivatives will fail to exist at: