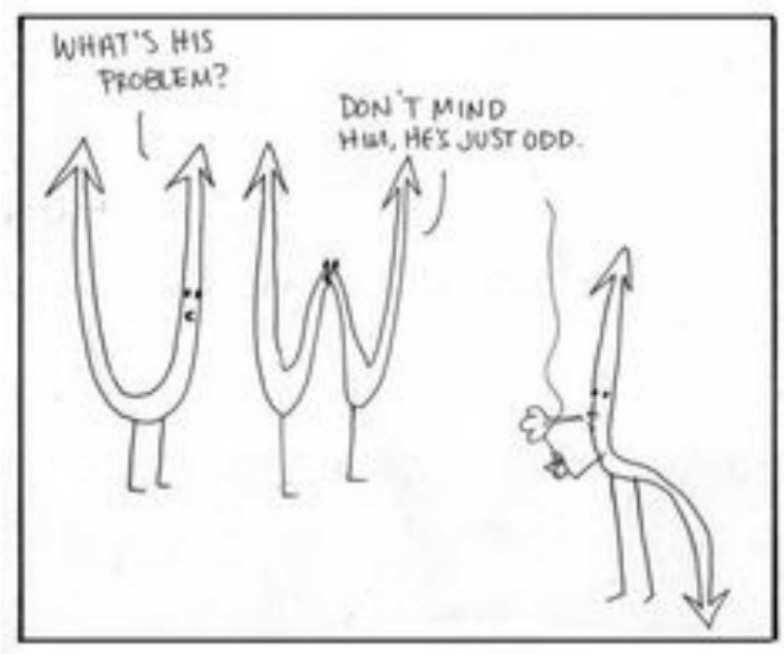
# 2.2 End Behavior, Real Zeros

Thursday 2/25/17 **I can:**

1. Describe the end behavior of polynomials by finding limits to infinity from a graph.
2. Find all real zeros of polynomial functions algebraically.
3. Identify multiplicities of real zeros & describe the effect on a graph.
4. Use the Intermediate Value Theorem to help locate real zeros of polynomial functions.



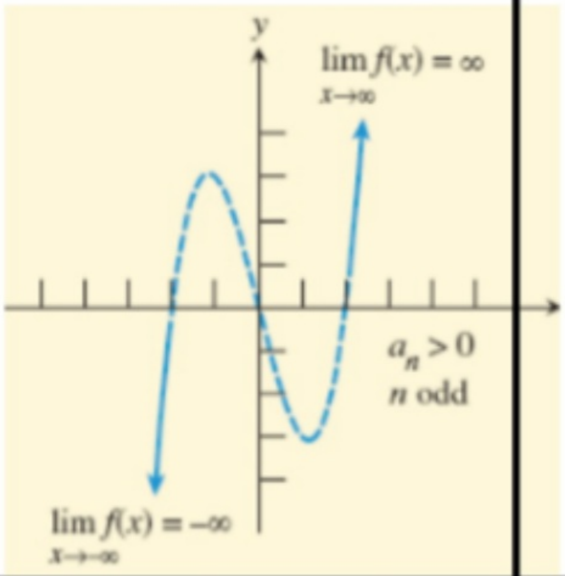
## What's the difference between a Power Function and a term of a Polynomial?

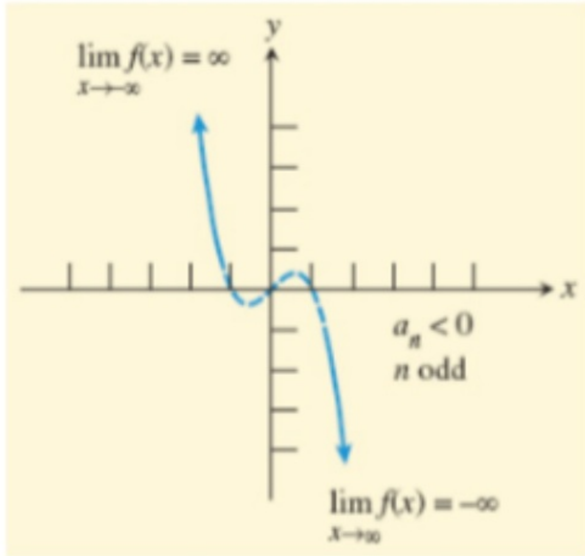
The power doesn't have to be a whole number in a Power Function.

It can be any number... positive or negative and need not be an integer, meaning fractions are allowed in the exponent.

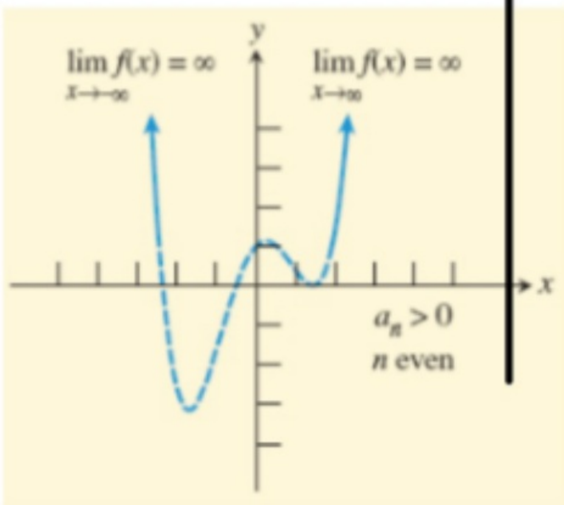
leading coefficient is positive | leading coefficient is negative

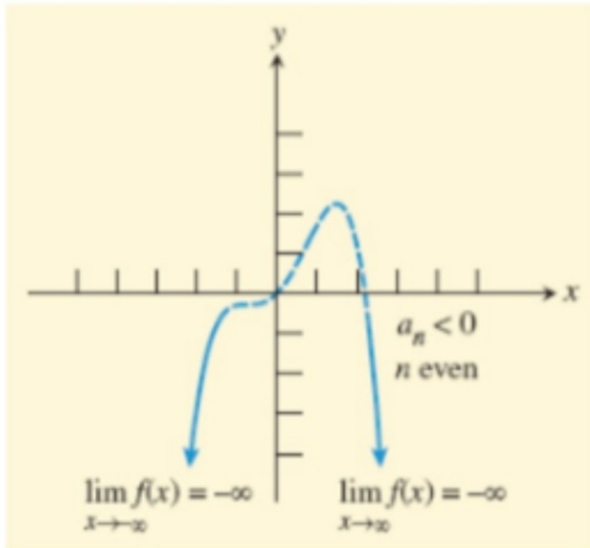
odd degree




even degree



Ex 1: Describe the left and right end behavior using limit notation:

**a.** $f(x) = -x^3 + 4x$ **b.** $f(x) = x^4 - 5x^2 + 4$ **c.** $f(x) = x^5 - x$

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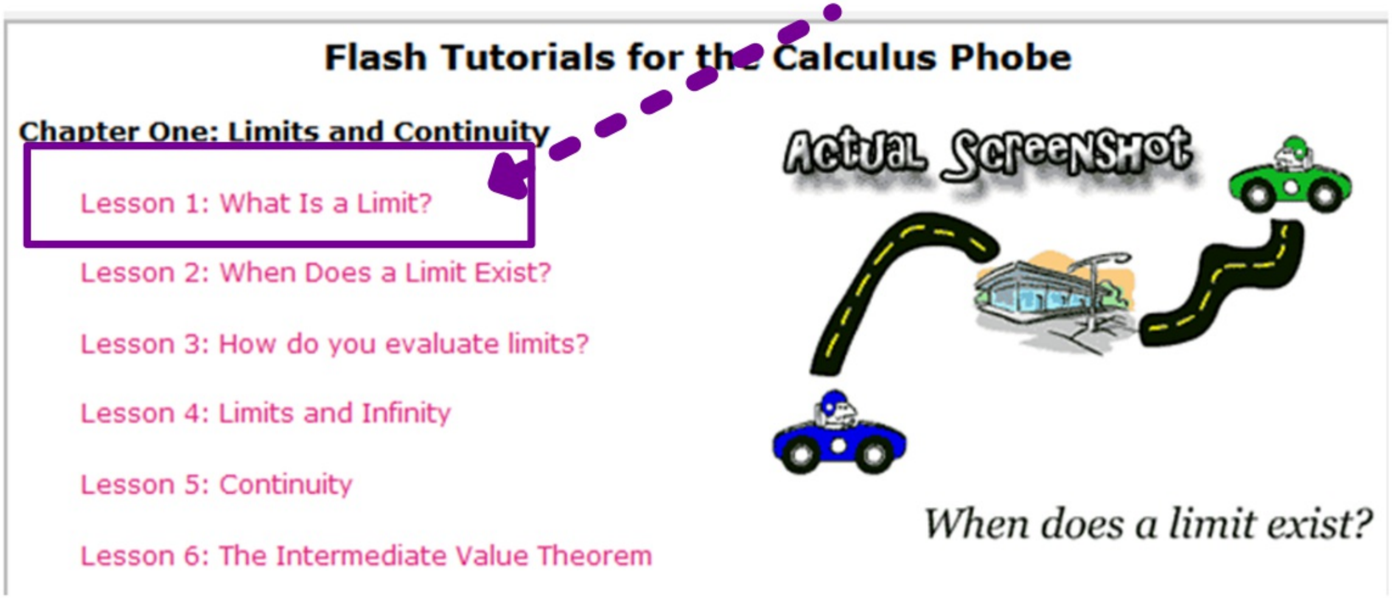

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# Key Concepts

## Zeros of Polynomial Functions

For a polynomial function of degree *n*, the following statements are true:

1. The function has at most *n* complex zeros (real & nonreal). Some of the zeros may be repeated.

**(Fundamental Theorem of Algebra)**

2. The graph of *f(x)* has at most *n-1* relative extrema (turning points)

## Real Zeros of Polynomial Functions

If $f$ is a polynomial function and $a$ is a real number, the following statements are equivalent.

1. $x = a$ is a *zero* of the function $f$.
2. $x = a$ is a *solution* of the polynomial equation $f(x) = 0$.
3. $(x - a)$ is a *factor* of the polynomial $f(x)$.
4. $(a, 0)$ is an *x-intercept* of the graph of $f$.

Ex 2: Identify the following features & find all real zeros algebraically of $f(x) = -2x^4 + 2x^2$

a) Name:

b) # of extrema:

c) Real zeros:

d) End behavior

Ex 3: Identify the following features & find all real zeros algebraically of $f(x) = 3x^3 - 12x^2 + 3x$

a) Name:

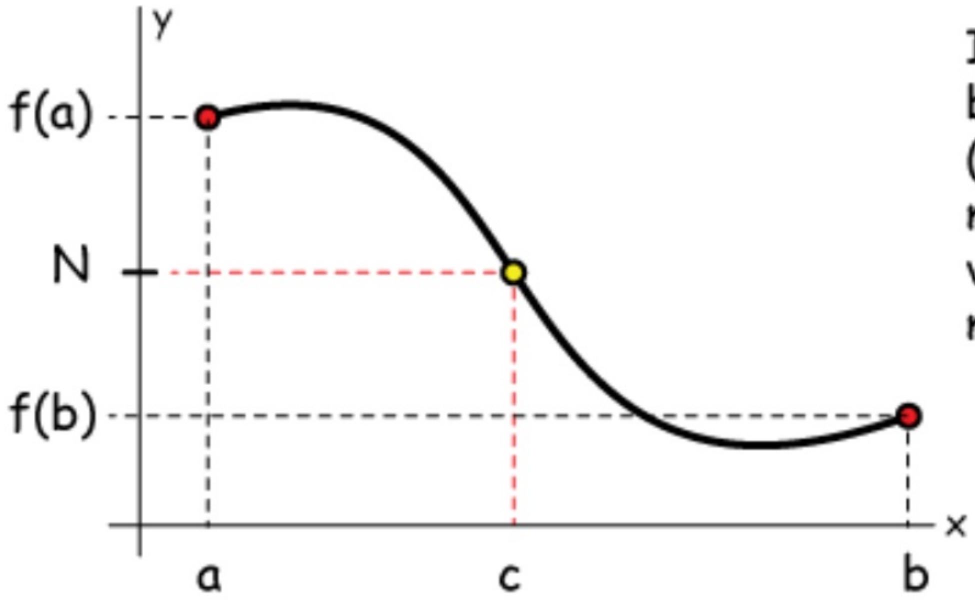
b) # of extrema:

c) Real zeros:

d) End behavior:

## The Intermediate Value Theorem

If **f** is a continuous function on the interval [a, b], and N is any number between f(a) and f(b), where f(a) ≠ f(b), then there is a number c in (a, b) such that f(c) = N.




If the graph of a function can be traced continuously from (a, f(a)) to (b, f(b)),then it must take on every intermediate value from f(a) to f(b) —possibly more than once — along the way.

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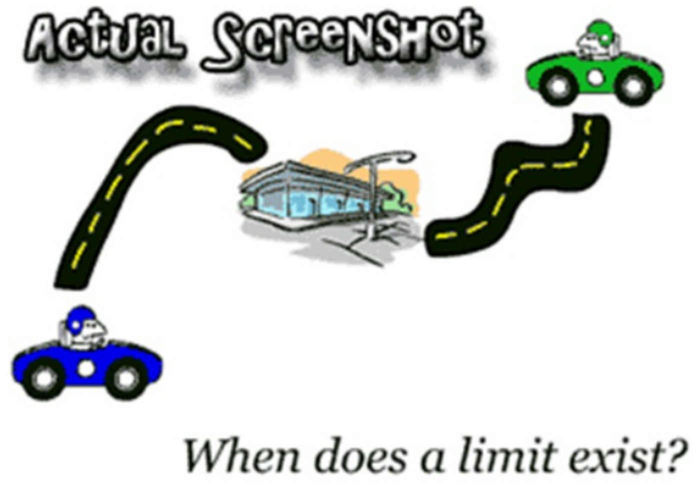
**Chapter One: Limits and Continuity**

- Lesson 1: What Is a Limit?
- Lesson 2: When Does a Limit Exist?
- Lesson 3: How do you evaluate limits?
- Lesson 4: Limits and Infinity
- Lesson 5: Continuity
- Lesson 6: The Intermediate Value Theorem



# Why is this important?

The Intermediate Value Theorem helps you locate the value of ___________. If you can find a value at x = a at which the polynomial function is ____________ and another value at x = b at which it is ______________, you can conclude that the function has at least one _____________ between these two values.

# What the function?!

| X | $Y_1$ |
|---|---|
| 0 | -3 |
| 1 | 1 |
| 2 | 5 |
| 3 | 9 |
| 4 | 13 |
| 5 | 17 |
| 6 | 21 |

X=6

Use the Intermediate Value Theorem to determine if there is a zero on the given interval.

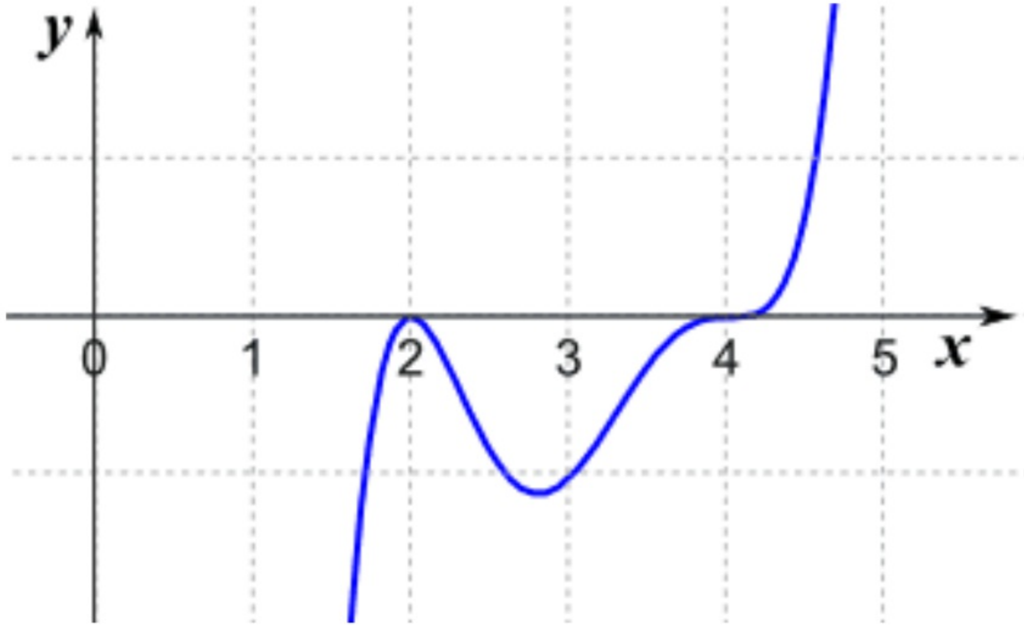
Ex 4: $f(x) = x^3 + 4x^2 - 5x + 9$

$-6 < x < -5$

Ex 5: $f(x) = x^3 + x^2 - 3x + 7$

$-5 < x < -2$

# Multiplicity

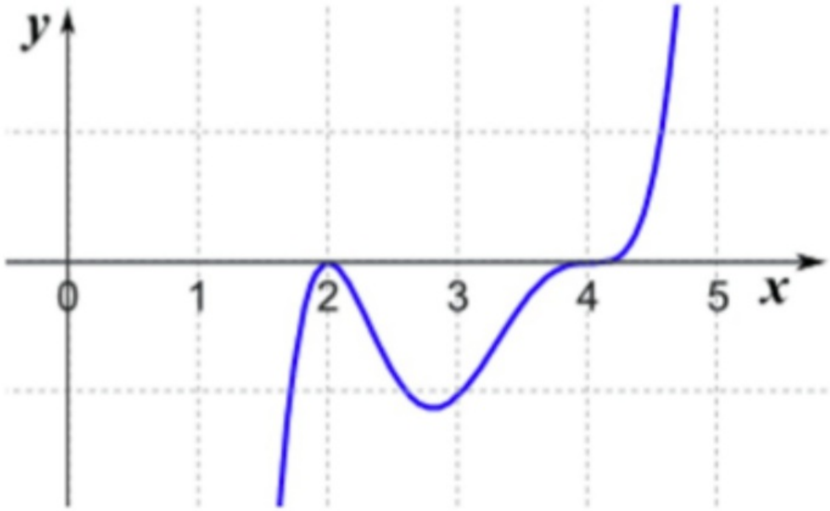



A repeated factor creates a repeated zero.

If the zero is a real number, then

an even multiplicity → the graph will BOUNCE at that *x*-intercept.

an odd multiplicty → the graph will CROSS at that *x*-intercept



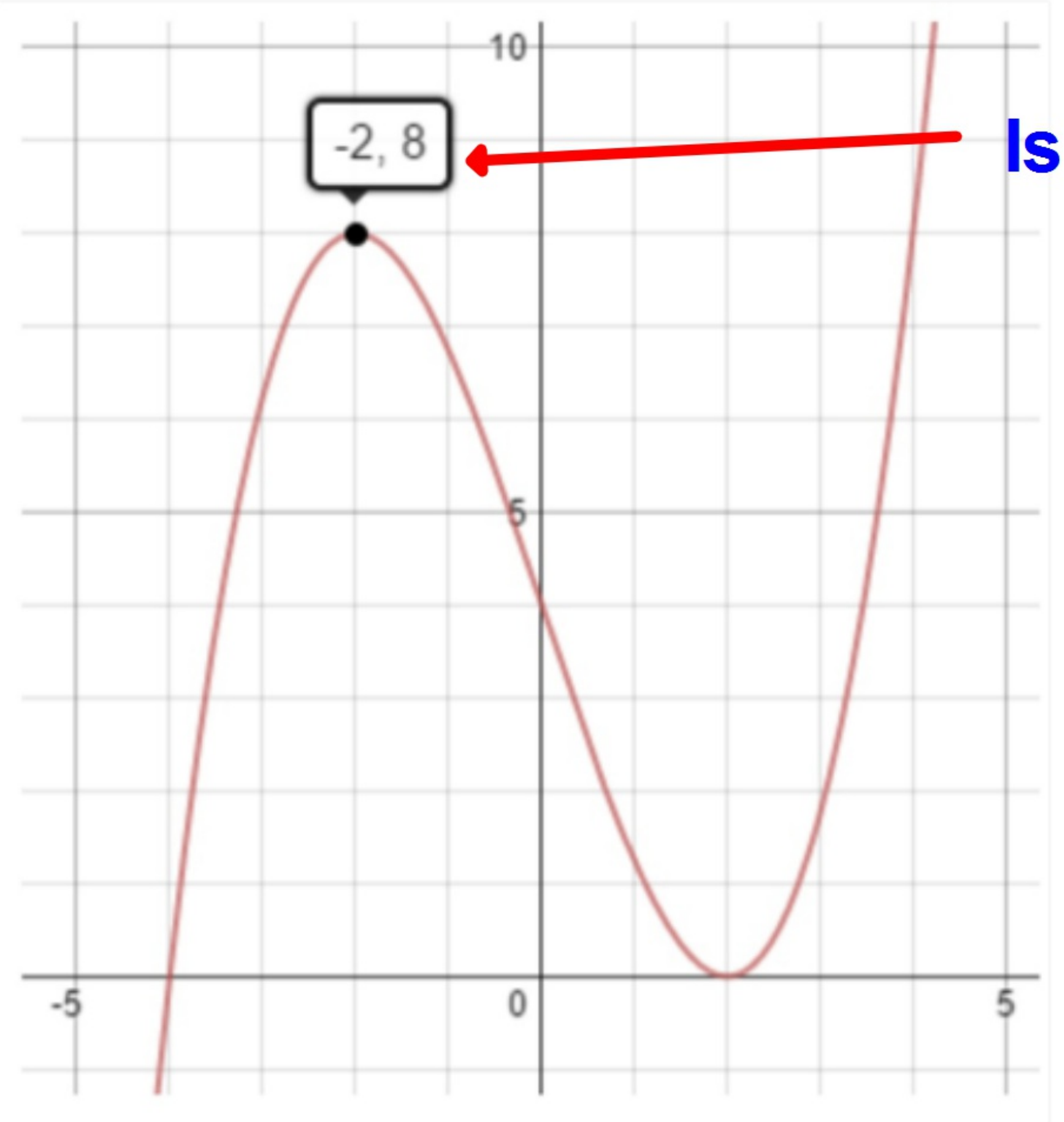
For each function below:

a) Name the zeros (ordered pairs)
b) Describe the multiplicity for every zero
c) Describe the end behavior using limit notation
d) Sketch a graph based on a-c

Ex 6: $g(x) = (x-2)^3 (x+1)^2$

$f(x) = ($

Ex 8: Write the equation for the polynomial shown




**Is this important?**