2.3 Differentiation Rules

$\frac{dy}{dx}$ is a **noun**. It means "the derivative of y with respect to x."

$\frac{d}{dx}$ is a **verb**. It means "take the derivative with respect to x" of the expression that follows.

The Constant Rule

The derivative of a constant function is 0.

Example 1:

Find the derivative of the following functions:

(a) $y = 8$ (b) $f(x) = 0$ (c) $s(t) = 3$ (d) $y = k\pi^2$, k is a constan

The Power(ful) Rule

If n is a real number and a is some constant in the function $f(x) = ax^n$, then $\frac{d}{dx}\left[ax^n\right] = anx^{n-1}$. Equivalently, $f'(x) = anx^{n-1}$.

Example 2:

Find the derivative of the following functions:

(a) $f(x) = 2x^3$

(b) $g(x) = \frac{\sqrt[3]{x}}{3}$

(c) $y = \frac{5}{3x^\pi}$

(d) $y = \frac{6}{\sqrt[5]{x^3}}$

Example 3:

If $f(x)=\frac{x^4}{2}$, find each of the following.

(a) $f'(-1)$

(b) $f'(0)$

(c) The x-coordinate where f has a slope of 128.

Example 4:

If $f(x) = 3x^2$

(a) find the equation of the tangent line at $x = -2$.

(b) find the equation of the normal line at $x = -2$.

(c) find the points where the normal line intersects the graph of $f(x) = 3x^2$.

Rewriting is very important when using the Power Rule.

Example 6:
Rewrite, evaluate (differentiate), and then simplify the following:

(a) $\frac{d}{dx}\left[\frac{5}{2x^{\sqrt{2}}}\right]$

(b) $\frac{d}{dx}\left[\frac{4}{(2x)^{3}}\right]$

(c) $\frac{d}{dt}\left[\frac{7t}{3\sqrt{t}}\right]$

(d) $\frac{d}{dm}\left[\frac{6}{(3m)^{-2}}\right]$

The Sum and Difference & Konstant Rules

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$ Addition/Subtraction Rule

$$\frac{d}{dx}\left[kf(x)\right] = k\frac{d}{dx}\left[f(x)\right]$$ Konstant Rule

Example 7:

Find the derivative of the following functions:

(a) $f(x) = x^3 - 4x + 5$

(b) $g(x) = -\frac{x^4}{2} + 2x^3 - 5x$

(c) $y = \dfrac{2x^3 - 3x^2 + 7x + 5}{2\sqrt{x}}$

(d) $y = x(3x+2)^2$

Example 8:

Find the **coordinates** (x, y) and **equations** of any horizontal tangents to the curve $y = x^4 - 2x^2 + 2$.

at any point of tangency, $x = c$, a function $f(x)$ and its tangent line $T(x)$ share two things:

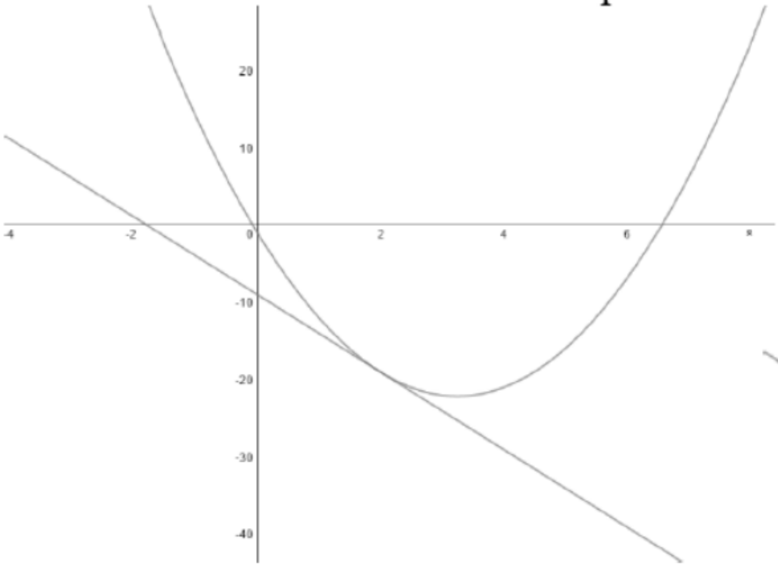
1. A *y*-value

2. A slope value

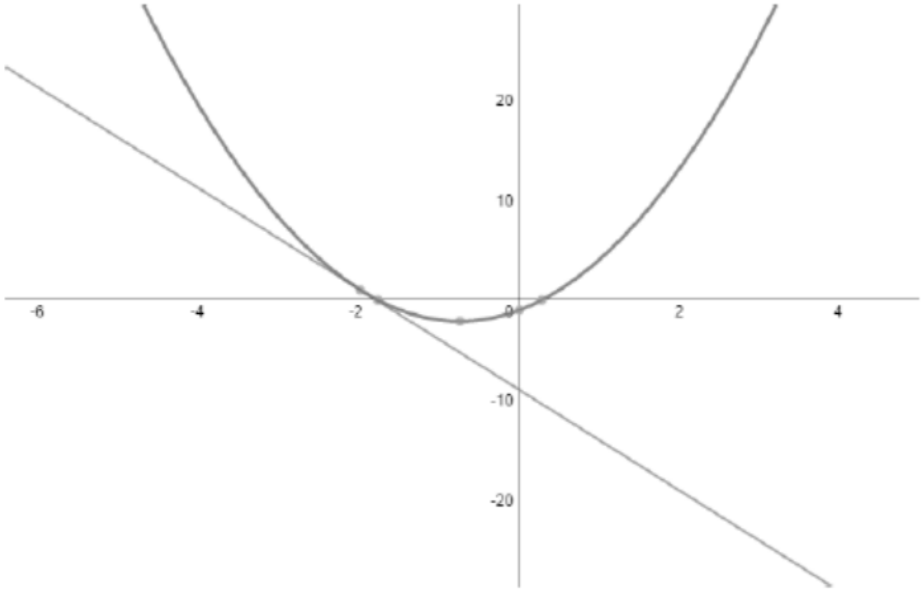
Example 9:

Find the value(s) of k such that the line $5x + y = -9$ is tangent to the graph of $f(x) = 2x^2 + kx - 1$.

Here are the solution curves to the problem above.



$$f(x) = 2x^2 - 13x - 1$$



$$f(x) = 2x^2 + 3x - 1$$