2.6 Implicit Differentiation

The equations below are equivalent, but are in different forms with respect to the dependent variable y.

Explicit form	**Implicit form**
$y = \frac{2}{x}$	$xy = 2$

Example 1:

Find the derivative, $\frac{dy}{dx}$, of $y = \frac{2}{x}$, an equation explicitly solved for y by taking the derivative of both sides with respect to x.

Example 2:

Find the derivative, $\frac{dy}{dx}$, of $xy = 2$, an equation NOT explicitly solved for y by taking the derivative of both sides with respect to x. Solve for $\frac{dy}{dx}$.

The method used to find $\frac{dy}{dx}$ in Example 2 is called **Implicit Differentiation**. What do you notice abou the implicit result compared to the result from Example 1? Which method was easier/faster?

$$\frac{d}{dx}[\text{Left Side}] = \frac{d}{dx}[\text{Right Side}]$$

Example 3:

For the equation $y = x + \sec(y)$, find $\frac{dy}{dx}$.

Example 4:

Find the slope of the graph of $y^3 + y^2 - 5y - x^2 = -4$ at $(1,-3)$ by finding $\frac{dy}{dx}$ first, then evaluating $\frac{dy}{dx}$ at $(1,-3)$

Example 5:

Find the slope of the graph of $y^3 + y^2 - 5y - x^2 = -4$ at $(1,-3)$ by differentiating then plugging in $(1,-3)$ **before** solving for $\frac{dy}{dx}$.

Notice that in Examples 4 and 5, because our derivative equation had both an x and a y in it, we needed the actual ordered pair (x, y) to evaluate $\frac{dy}{dx}$. Often when only an x-value is given, and not an ordered pair, it is sign that our given equation can be explicitly solved for a single equation of y, like in Examples 1 and 2, but not always.

Example 6:

Find $\frac{dy}{dx}$ at $x=1$ for the equation $y^2 + x = 2xy$

When your derivative $\frac{dy}{dx}$ is a quotient of two variable expressions, then

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- Horizontal tangents occur when $\frac{dy}{dx} = \frac{0}{\neq 0}$
- Vertical tangents occur when $\frac{dy}{dx} = \frac{\neq 0}{0}$
- No tangent line exists when $\frac{dy}{dx} = \frac{0}{0}$ (these points must be thrown out)

Example 7:

The graph of the equation $x^2 + y^2 = 4$ is a circle centered at the origin with a radius of four.

(a) Sketch the graph of the equation

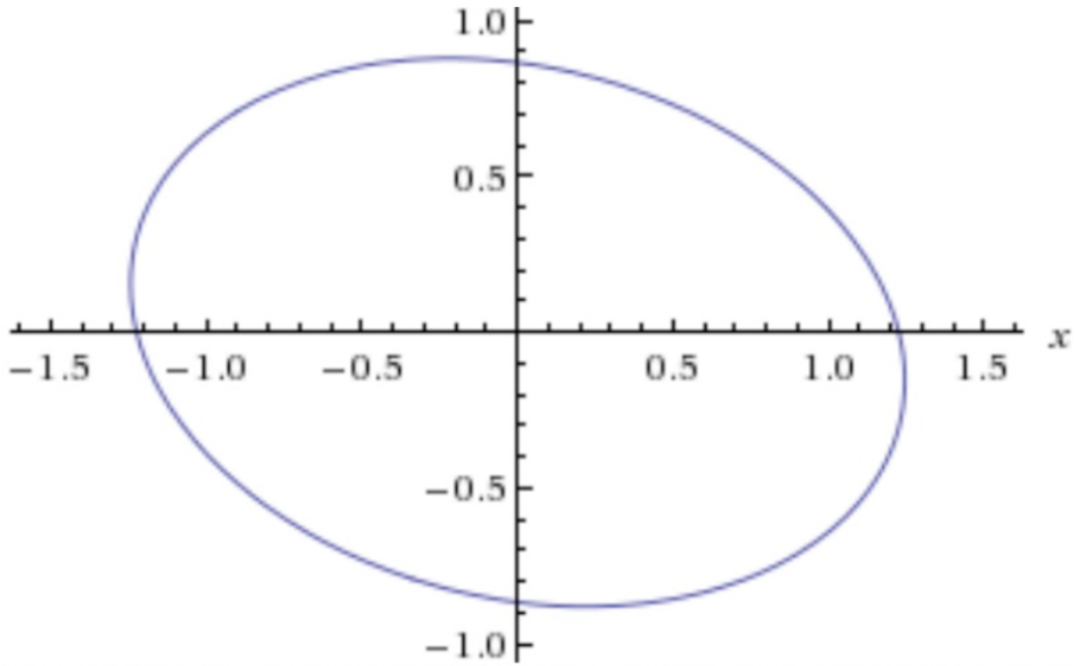
(a) find $\frac{dy}{dx}$.

(b) Using calculus, verify that the graph has horizontal tangents at the point $(0,2)$ and $(0,-2)$.

(c) Using calculus, verify that the graph has vertical tangents at the point $(2,0)$ and $(-2,0)$.

(d) At what value(s) of x is the slope of the graph $\frac{3}{4}$?

Example 8:



The equation for the graph of the rotated ellipse shown above is $2x^2 + xy + 4y^2 = 3$,

(a) Determine the x-value(s) of any horizontal tangent lines to the graph of $2x^2 + xy + 4y^2 = 3$.

(b) Determine the y-value(s) of any vertical tangent lines to the graph of $2x^2 + xy + 4y^2 = 3$.

Example 9:

Determine the equation of the tangent line of $3\left(x^2 + y^2\right)^2 = 100xy$ at the point $(3,1)$

Example 10:

Find $\frac{d^2y}{dx^2}$ **as a function of** x **and** y if $2x^3-3y^2=8$.