2.7 Special Derivatives

Exponentials

Example 1:

Sketch the graph of $f(x)=e^x$, then, on the same set of axes, sketch a possible graph of $f'(x)$. What do you notice? Confirm by sketching $f'(x)$ using your calculator's NDERIV capability.

Derivative of e^x

$\frac{d}{dx}\left[e^x\right]=e^x$. If u is a differentiable function of x, then $\frac{d}{dx}\left[e^u\right]=e^u \cdot u'$ (Chain Rule)

Example 2:

Find $\frac{dy}{dx}$ if $y=e^{\left(x+x^2\right)}$

Example 3:

Using your calculator, graph $f(x)=2^x$ and $f'(x)$ using NDERIV. What do you notice? Do the same for $g(x)=5^x$ and $g'(x)$.

General Derivative of b^x

$\frac{d}{dx}\left[b^x\right]=b^x \cdot \ln b$. If u is a differentiable function of x, then $\frac{d}{dx}\left[b^u\right]=b^u \cdot \ln b \cdot u'$ (Chain Rule)

Example 4:

At what point on the graph of the function $y=2^t-3$ does the tangent line have a slope of 2?

Logarithms

$$b^x = y$$
$$\Leftrightarrow$$
$$\log_b y = x$$

Example 1:

Evaluate the following quickly:

(a) $\log_2 8$

(b) $\log_3 \frac{1}{81}$

(c) $\log_{16} 4$

MEMORIZE

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\frac{d}{dx}\left[\ln|x|\right] = \frac{1}{x}$ $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$ (chain rule)

Find the derivative of each of the following. **Remember to simplify early and often, ESPECIALLY WHEN YOU HAVE LOGS!**

(a) $h(x) = \cos(\ln x)$ (b) $y = \ln(1 + \ln x)$ (c) $f(u) = \ln\sqrt{\frac{3u+2}{3u-2}}$

MEMORIZE

$$\frac{d}{dx}[\log_b x] = \frac{1}{x}\cdot\frac{1}{\ln b} = \frac{1}{x\ln b} \qquad \frac{d}{dx}\left[\log_b |x|\right] = \frac{1}{x\ln b} \qquad \frac{d}{dx}[\log_b u] = \frac{1}{u\ln b}\cdot u' \text{ (chain}$$

If $f(x) = \left(\log_3\left(5 - x^4\right)\right)^2$, find $f'(x)$.

Inverse Trig Derivatives

The Derivatives of the six Inverse Trig Functions (MEMORIZE (if you haven't already))

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\operatorname{arc\,cot} x] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}[\operatorname{arc\,sec} x] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[\operatorname{arc\,csc} x] = \frac{-1}{|x|\sqrt{x^2-1}}$$

Find the equation of the tangent line to the graph of $y = \cot^{-1} x$ at $x = 1$

Differentiate:

(a) $\frac{d}{dx}\left[4x+\arcsin\left(2x^2\right)\right]$

(b) $\frac{d}{dx}\left[x\arctan\sqrt{x}\right]$

(c) $\frac{d}{dx}\left[\frac{\operatorname{arcsec}(\sin 3x)}{5}\right]$

Differentiate $y=\arcsin x+x\sqrt{1-x^2}$, then simplify to a single term.

Find the coordinate of any horizontal **tangent** and find the equation of any horizontal **asymptotes** of $g(x) = (\arctan x)^2$