UNIT 3: Derivative Applications

This is where things get exciting!!!!

3.1 Extrema on an Interval

Definition: (Absolute/Global) Extrema/Extreme *y*-values

If f is a function on an interval *I*, then $y = f(c)$ is the

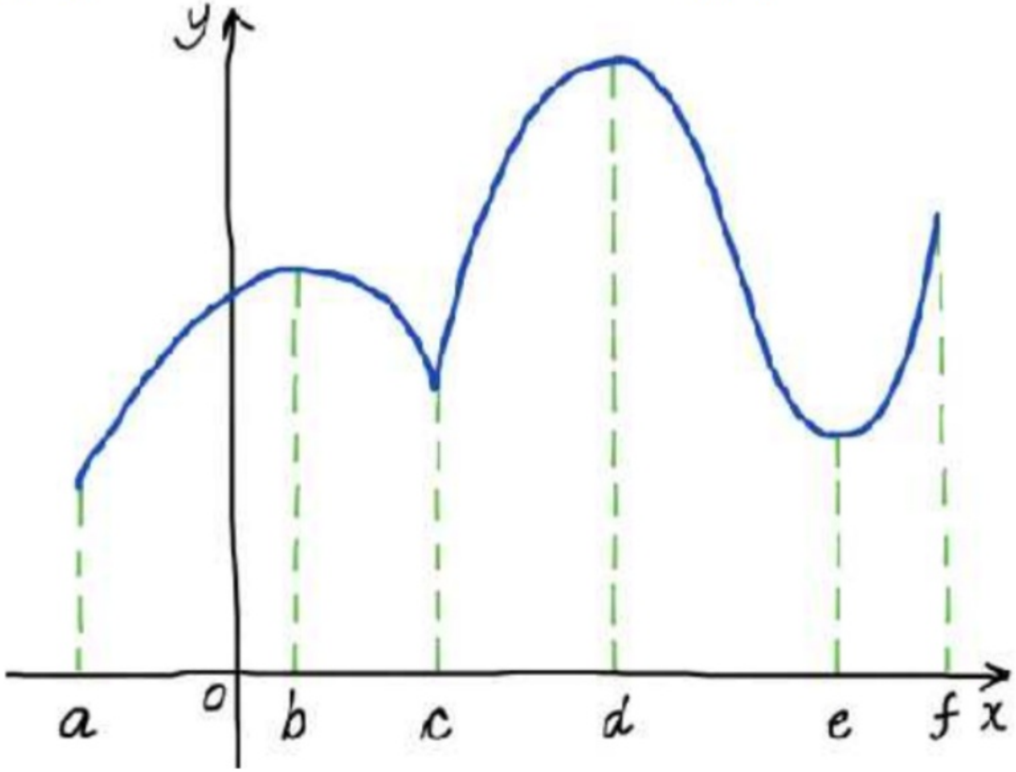
I. (Absolute/Global) **Maximum** on *I*, IFF $f(c) \geq f(x)$ for all x in *I*.

II. (Absolute/Global) **Minimum** on *I*, IFF $f(c) \leq f(x)$ for all x in *I*.

Notice we cannot say that the maximum is the *y*-value that is the BIGGEST on an interval, but rather we must say it's the *y*-value for which there are none bigger. Similarly with the minimum.

Example 1:

The graph of $g(x)$ is given below. Determine the extrema of $g(x)$ on the interval $x \in [a, f]$.



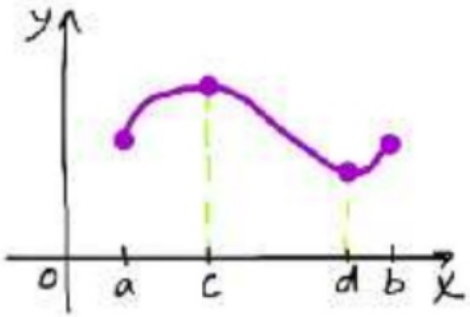
Example 2:
Sketch the following functions on the given interval, then, determine the extrema, if they exist.

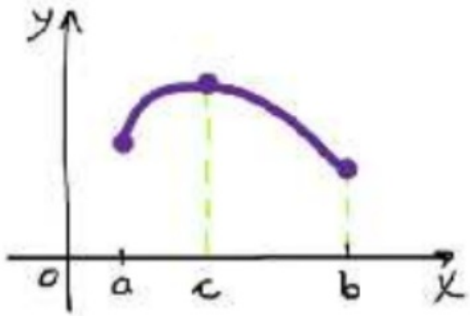
(a) $f(x) = x^2 + 1$ on $[-1, 2]$ (b) $f(x) = x^2 + 1$ on $(-1, 2)$ (c) $f(x) = \begin{cases} x^2 + 1, & x \neq 0 \\ 2, & x = 0 \end{cases}$ on $[-1, 2]$

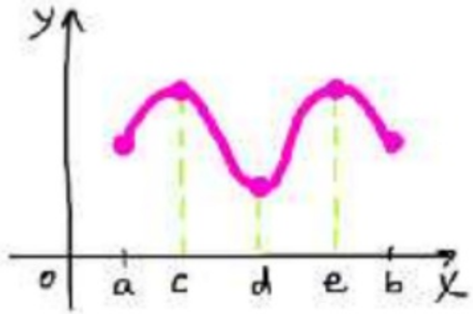
The Extreme Value Theorem (EVT)

If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval $[a, b]$.

Here are some examples of functions on $[a, b]$ where the EVT applies.







Definition: Critical Value

A **critical value** of a function f is a value $x = c$ in the domain of f such that either

$$f'(c) = 0 \text{ or } f'(c) = DNE$$

If $x = c$ is a critical value, then $(c, f(c))$ is a critical point.

Theorem

(Absolute/Global) Extrema can only occur on at a **critical value** OR at an **endpoint** of an interval.

Definition: Relative/Local Extrema

If there exists an **open** interval containing $x = c$, then

I. If immediately to the left of $x = c$ and immediately to the right of $x = c$ there are no *y*-values **greater** than $f(c)$, then $f(c)$ is a **relative/local maximum** of *f*.

II. If immediately to the left of $x = c$ and immediately to the right of $x = c$ there are no *y*-values **smaller** than $f(c)$, then $f(c)$ is a **relative/local minimum** of *f*.

Theorem

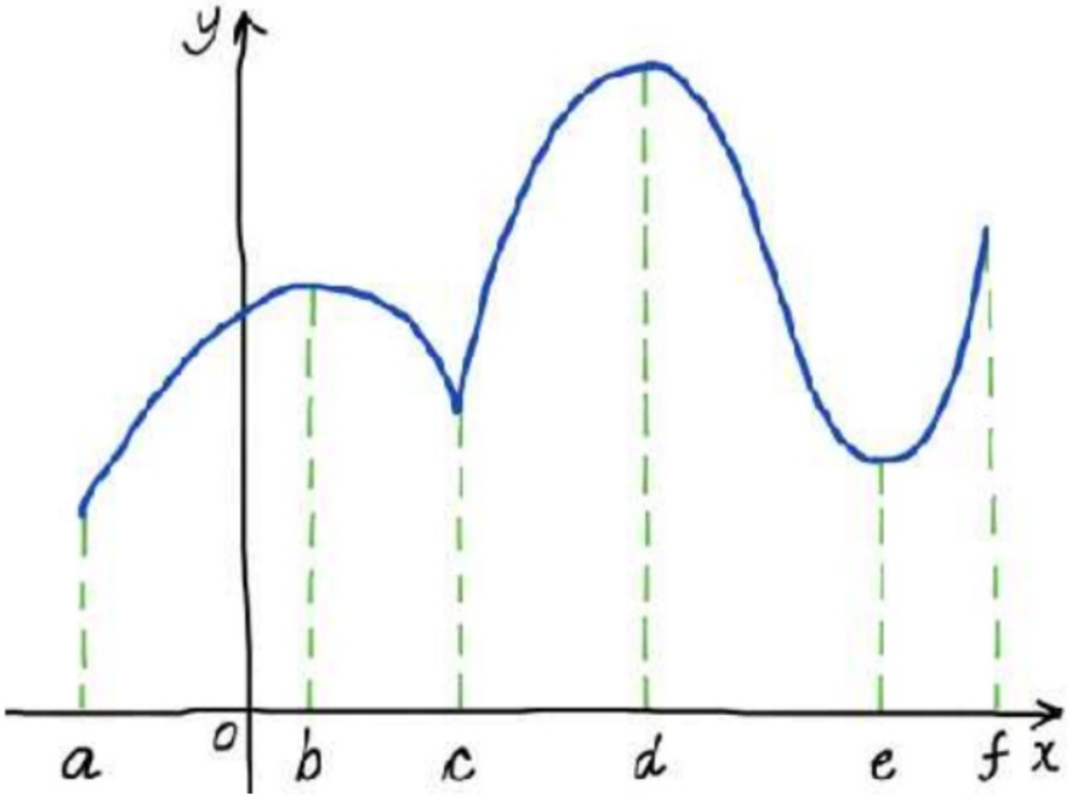
Relative/Local Extrema can only occur on at a **critical value** on an **OPEN** interval.

NOTE I: This theorem does NOT say that there is a relative/local extreme value at every critical value (for example, $f(x) = x^3$ at $x = 0$), but rather, every critical value is the potential, prospective, possible location of a relative/local extreme value.

***** NOTE II: Relative/Local extrema CANNOT occur at an endpoint of an interval. They cannot live at the end of a cul-de-sac, but rather along a through street.***

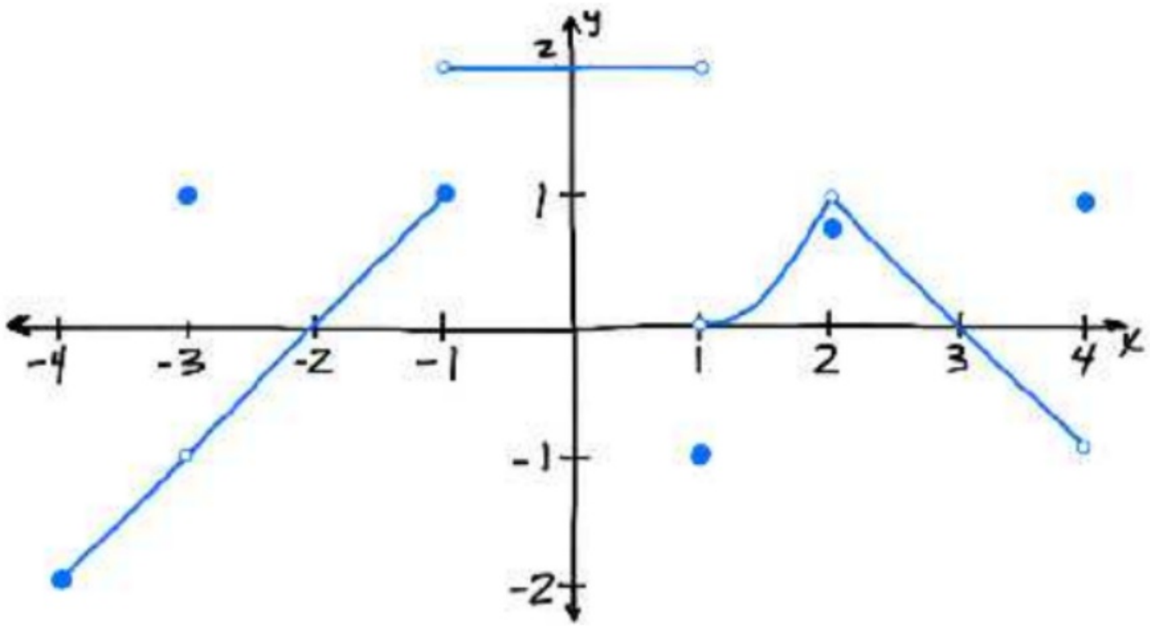
Example 4:

Identify all the critical values of the graph of $g(x)$ shown below, then determine whether a local max, local min, or neither occur at that critical value.



Example 5:

The graph of $h(x)$ is given below. Find all the critical values of $h(x)$ on the interval $[-4, 4]$, then determine if a local max, local min, or neither occur at each.



Example 6:

Find the **domain** and the **critical values** of each of the following functions:

(a) $f(x)=\frac{1}{2}x^4-x^3-x^2+2$ (b) $g(x)=2-|x-4|$ (c) $f(x)=x^{3/5}(4-x)$.

Closed Interval Method for finding Extrema

To find absolute extrema of a continuous function $f(x)$ on a closed interval $[a,b]$.

1. Identify the endpoints.
2. Identify any critical values in (a,b)—**verify they are in the <u>domain</u> & the <u>specified interval</u>!!**
3. Find the function values, $f(x)$, at both endpoints and at all critical values in (a,b).
4. The largest value will be the max. The smallest value will be the min.
5. Answer the question asked in a complete sentence.

Example 7:

Determine if the EVT applies. If so, find the extrema of $f(x) = 3x^4 - 12x^3$ on the interval $[-1, 2]$.

Example 8:

Determine if the EVT applies. If so, find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval $[-8, 1]$

Example 9:

Determine if the EVT applies. If so, find the extrema of $f(x) = 2\sin x - \cos 2x$ on $\left[0, \frac{3\pi}{2}\right]$.

What do we do if we want to find extrema, the EVT does not apply, no interval is given? Use all the tools at your disposal, including sketching a graph.

Example 10:
Find the extrema of each of the following functions over their domain by sketching their graphs:

(a) $f(x) = \dfrac{1}{\sqrt{4-x^2}}$

(b) $f(x) = \begin{cases} 5-2x^2, & x \le 1 \\ x+2, & x > 1 \end{cases}$

Sometimes we need to use our calculator's number-crunching ability to **solve equations by finding *x*-intercepts for us.**

Example 11:
Using your calculator's equation solving capability (not just its max/min finding ablity), find the extrema of the $f(x) = xe^{-x} - 2x$ on the interval $[-1,1]$. Be sure to show the equation you're solving and you justification via the Closed Interval Argument.