3.1 Extrema on an Interval Practice

Show all work. No calculator, except on problem #6.

1. Let f be the functioned defined on $[-1,2]$ by $f(x)=3x^{2/3}-2x$.

   (a) Find the domain of $f(x)$.

   (b) Find $f'(x)$ and the domain of $f'(x)$.

   (c) Determine if the EVT applies to $f(x)$ on the given interval. If it does, find the max and min values guaranteed by the theorem. Justify using the Closed Interval Argument. Show the work that leads to your answer.

2. Sketch the graph of a function f that is continuous on $[1,5]$ and has an absolute minimum at $x=1$, an absolute maximum at $x=5$, a local maximum at $x=2$, and a local minimum at $x=4$. Answers will vary.

3. Sketch the graph of f and use your sketch to find the absolute and local extrema of f on the indicated domains.

   (a) $f(x)=\begin{cases}1-x, & 0\le x<2\\ 2x-4, & 2\le x\le 3\end{cases}$ (b) $f(x)=\begin{cases}x^2, & -1\le x<0\\ 2-x^2, & 0\le x\le 1\end{cases}$

4. Find the critical values of the functions over their domains. **Remember that a critical value MUST be in the domain of the function, though it may not be in the domain of that function's derivative function. Also, be on CUSP ALERT!**

   (a) $x(t)=3t^4+4t^3-6t^2$ (b) $f(z)=\dfrac{z+1}{z^2+z+1}$ (c) $g(t)=5t^{2/3}+t^{5/3}$

5. Find the absolute extrema of f on the given interval.

(a) $f(x)=2x^3-3x^2-12x+1,\ [-2,3]$ (b) $f(x)=\left(x^2-1\right)^3,\ [-1,2]$

(c) $f(t)=\sqrt[3]{t}\,(8-t),\ [0,8]$ (d) $f(x)=\sin x+\cos x,\ \left[0,\frac{\pi}{3}\right]$

6. (Calculator Permitted) Using your calculator's equation solving capability (not just its max/min finding ablity), find the extrema of the $f(x)=\dfrac{\ln x-e^x}{x}$ on the interval $[1,3]$. Be sure to show the equation you're solving and you justification via the Closed Interval Argument.

____ 8. Find all the critical values, $x=c$, of the function $g(x)=5x+\sin 5x$ in $(0,\infty)$, where $n=0,\ 1,\ 2,\ \ldots$.

(A) $c=\dfrac{3\pi}{5}n+\dfrac{\pi}{5}$ (B) $c=\dfrac{\pi}{5}n$ (C) $c=\dfrac{2\pi}{5}n+\dfrac{\pi}{5}$ (D) $c=\dfrac{4\pi}{5}n+\dfrac{\pi}{5}$ (E) $c=\dfrac{\pi}{5}n+\dfrac{\pi}{5}$

___ 9. Determine the absolute maximum value of $f(x)=\dfrac{5+2x}{x^2+14}$ on the interval $[-2,4]$.

(A) $\dfrac{1}{18}$ (B) $\dfrac{13}{30}$ (C) $\dfrac{8}{7}$ (D) $\dfrac{1}{2}$ (E) None

___ 10. Find all the critical values of f when $f(x)=x^{4/5}(x-5)^2$. Be sure to factor out least powers after differentiating.

(A) $0,\ \dfrac{5}{7}$ (B) $\dfrac{10}{7},\ 5$ (C) $\dfrac{5}{7},\ 5$ (D) $0,\ \dfrac{5}{7},\ 5$ (E) $0,\ \dfrac{10}{7}$ (F) $0,\ \dfrac{10}{7},\ 5$