3.2 Rolle's and MVT

e's Theorem (circa 1691)

'be a function that satisfies the following three hypothesis:

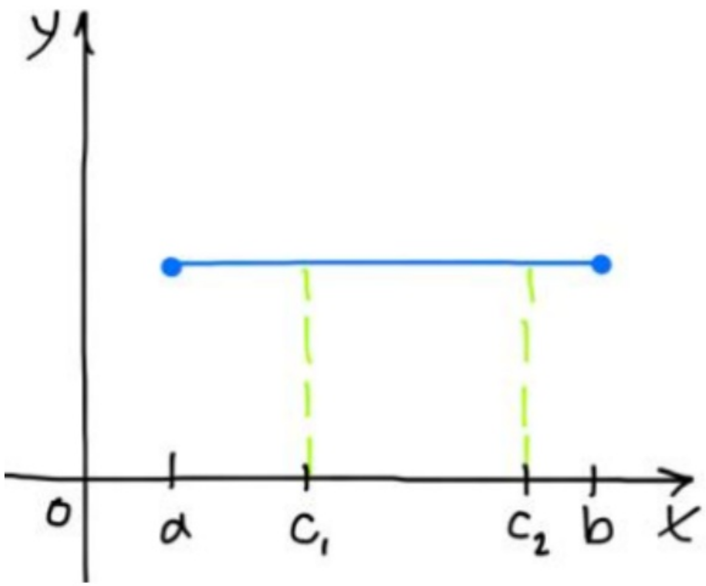
. f is continuous on the closed interval $[a,b]$.

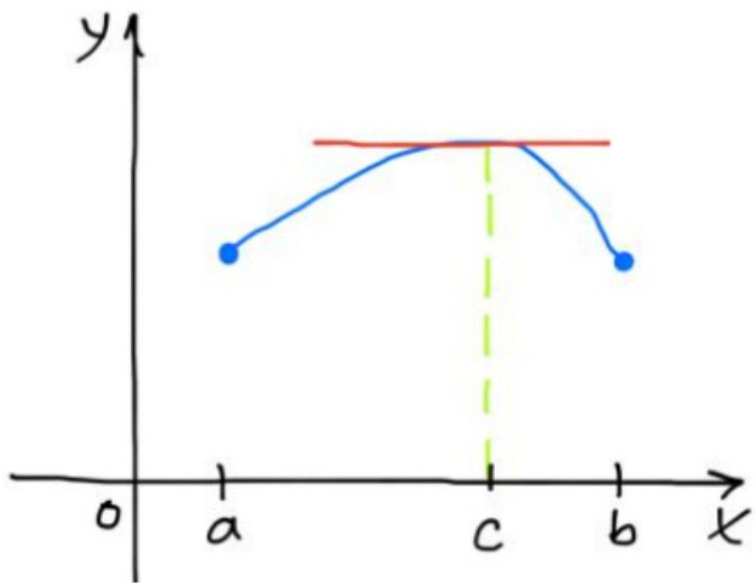
. f is differentiable on the open interval (a,b).

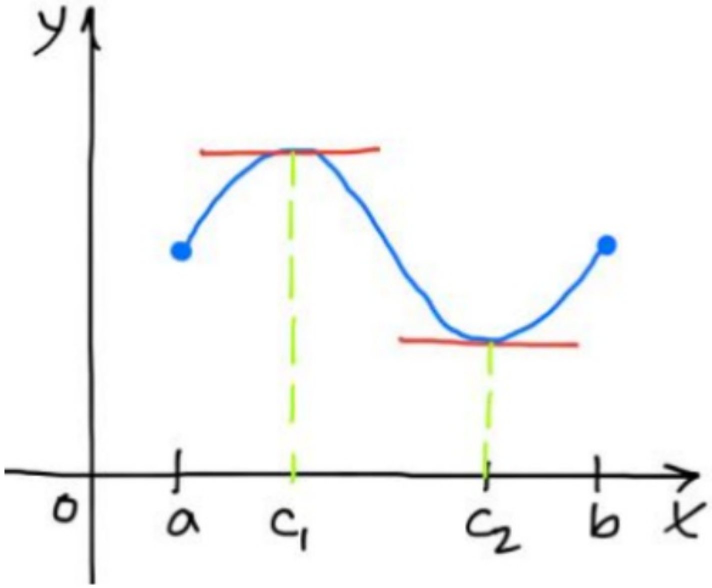
. $f(a) = f(b)$

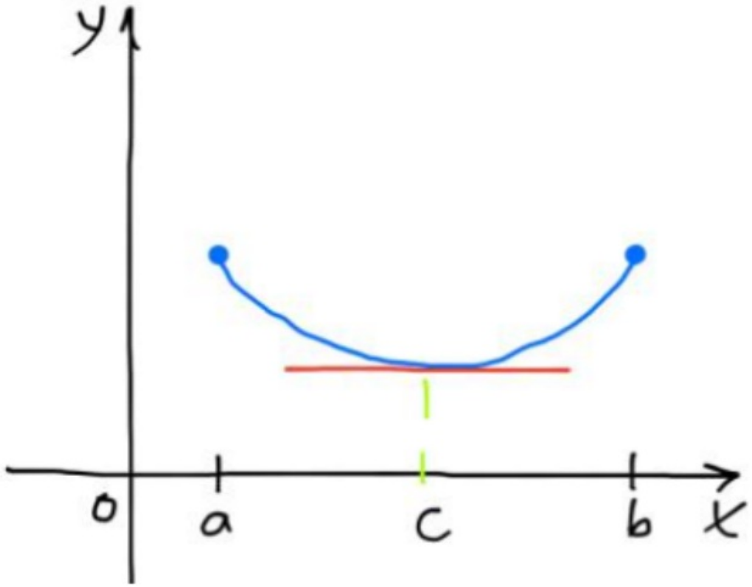
there is a number $x = c$ in (a,b) such that $f'(c) = 0$

are some functions that satisfy all three hypotheses.









nple 1:

$f(x) = x^4 - 2x^2$ on $[-2,2]$, determine if Rolle's Theorem applies. If so, find the value(s) of x inteed by the theorem.

Mean Value Theorem:

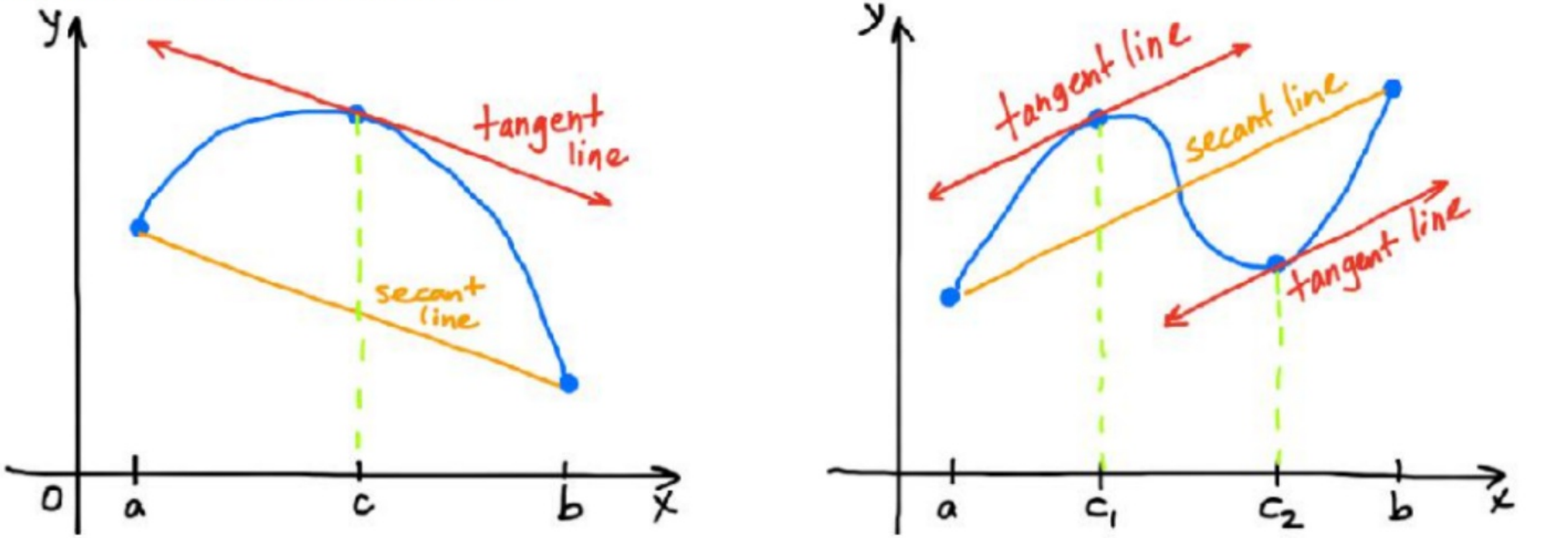
f be a function that satisfies the following two hypothesis:

1. f is continuous on the closed interval $[a,b]$.
2. f is differentiable on the open interval (a,b).

en there is a number $x=c$ in (a,b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

$$f(b)-f(a)=f'(c)(b-a)$$

Here's the graphical implication.



e slopes of the tangent line(s) and the slope of the secant line on the interval are the same!
e instantaneous rate of change and the average rate of change on the interval are the same!!

ple 2:

nine if the MVT applies to $f(x)=x^3-x$ on $[0,2]$, if so, find the value(s) guaranteed by the m.

ple 3:

he help of your calculator's ability to graphically find zeros, determine all the numbers c wl the conclusion of the Mean Value Theorem for the function $f(x)=x^3+2x^2-x$ on $[-1,2$

nportant to remember the interval in which you're working. Remember that both Rolle's Theor
ne MVT guarantee at least one value strictly on the OPEN interval.

nple 4:

mine if the MVT applies to $f(x) = x^3 - 3x^2 + 2x$ on the interval $[0,3]$. If so, find the values
nteed by the MVT.

ıple 5:

ıe following functions, determine if the MVT applies. If so, find the value of c guaranteed by t
ɜm. If not, specifically state why the theorem does not apply.

$(x) = \frac{x+5}{x-1}$ on $[-3,5]$

(b) $g(x) = x^{2/3}$ on $[-3,3]$

nple 7:

$f(x)$ be a function that is differentiable for all x.

uppose that $f(0)=-3$ and $f'(x)\leq 5$ for all values of x. How large can $f(2)$ possibly b

uppose that $f(5)=2$ and $f'(x)\geq -3$ for all x, what is the largest possible value of $f(1)$?

xample 6:

he calculus cops have set up their elaborate speed trap on a usy, stretchy stretch of road. A suspected pumpkin farmer who ses the road daily to haul his pumpkins to market is suspected of hronic speeding well beyond the posted limit of 55 mph. The alculus cops aim to finally ticket this unlawful transporter of easonal gourds. The calculus cops set up 5 miles apart from ach other, each parked furtively behind a piece of scenery. alculus cop A spots the farmer with his jalopy loaded down vith would-be jack-o-lanterns. As the farmer passes cop A, he is locked at a paltry 50 mph. Cop A could have sworn the farmer vaves at him as he drives by. Cop A immediately radios calculus op B 5 miles down the road, whereby cop A starts his timer. Four minutes later, cop B clocks the far

cruising by at only 55 mph. He clearly sees the farmer wave at him with a giant grin t would make a jack-o-lantern jealous. After a quick calculation on his field-issued TI- calculator, he pulls out with his lights on to issue a speeding ticket to the ornery pump farmer. For what speed can calculus cop B ticket the farmer? Would this ticket hold u a court of law? Why or why not?