# 3.5 Optimization

In this unit, we will be looking for **Absolute Extrema**, but we won't always have endpoints.

Here's what the Greek youth used to do for recreation before it was cool to use (Twitter/Facebook/Instagram . . . )

**Example 1:**
Find two positive numbers whose sum is 60 and whose product of one times the square of the other is a maximum. Don't get your toga in a knot!!

**Example 2**:
An 8-inch long pipe cleaner (used to clean pipes and form rectangles for a calculus class example) is bent into the shape of a rectangle. What dimensions of the rectangle will produce the rectangle with maximum area?

**Example 3:**

A ladybug farmer has 500 inches of fencing and wants to fence off a rectangular field that

borders on a straight river (to enclose his grazing ladybugs). He needs no fence along the river (ladybugs can't swim, and he has clipped their wings). What are the dimensions of the field that has the largest grazing area for his hungry ladybugs?

**Example 4:**

The same ladybug farmer has purchase some expensive, extraordinary diva ladybugs who

require ***exactly*** 10,000 square inches of grazing in order to be at their optimal "ladybug-like" state. What is the least amount of fencing required to get these diva ladybugs at their optimal state if the farmer is still allowed to build along the very straight river?? (Don't think diva lady bugs can swim and don't still have their wings clipped).

**Example 5:**

Find the point on the curve $y = x^2$ closest to $(3,0)$.

**WAYS TO JUSTIFY AN ABSOLUTE EXTREMA When you have relevant *ENDPOINTS***

**Method 1**: Closed interval argument (EVT) **When you have relevant *ENDPOINTS***

We can play the "biggest *y*-value" game with Team Endpoint and Team Critical Value. This sure-fire method requires finite endpoints and a continuous function over the relevant interval. This method will not work every time, because some examples may have at least one infinite (unbounded) endpoint. Also, once we reduce the primary equation down to a single variable, many equations will no longer be continuous. Alas, the method fails.

This works **when you have relevant *ENDPOINTS***

**Example 6:**

Find the point on the curve $y = \cos x$ closest to the point $(0,0)$.

**Example 7:**
Find the dimensions of the largest rectangle that can be inscribed under the curve $y = 16 - x^2$ in the first quadrant.

Steps to a successful Optimization Problem:

1. Read the problem carefully
2. Draw a picture. Label unknowns as variables. Label constants as numbers. IT IS EASIER TO LABEL SMALL PARTS AS SINGLE VARIABLES SO THAT YOU CAN AVOID FRACTIONS AND/OR SUBTRACTING.
3. Write a primary equation for the quantity to be optimized.
4. Identify the limiting factor/constraint and write a secondary equation (sometimes) involving this constraint.
5. Solve the secondary equation for any convenient variable and plug it into the primary equation to establish an equation for the optimal quantity in terms of a single variable.
6. Simplify the primary equation and think about a **relevant/feasible domain**.
7. Differentiate and find critical values within the relevant domain.
8. Determine the Absolute Extrema and JUSTIFY.
9. Answer the question in a complete sentence using appropriate units.
10. Smile ☺

**WAYS TO JUSTIFY AN ABSOLUTE EXTREMA (in absence of endpoints)**

**Method 2**: $1^{st}$ Derivative Test for Relative Extrema modified for Absolute Extrema
We will use this method when we don't have both endpoints at a closed interval, but instead have either a half-open interval or open interval. It does require a continuous function, though.

**Method 3**: $2^{nd}$ Derivative Test for Relative Extrema modified for Absolute Extrema
This method is preferred to method 2 when the $2^{nd}$ derivative is easy to obtain.

**In either case, it is important to show the test works FOR ALL values in the relevant domain. This transforms each test from a local argument to a global argument.

**Example 8:**
An open-top box is to be constructed from a square piece of cardboard, 3 feet wide, by cutting out a square from each corner and bending up the sides. What is the largest volume that the box can have?