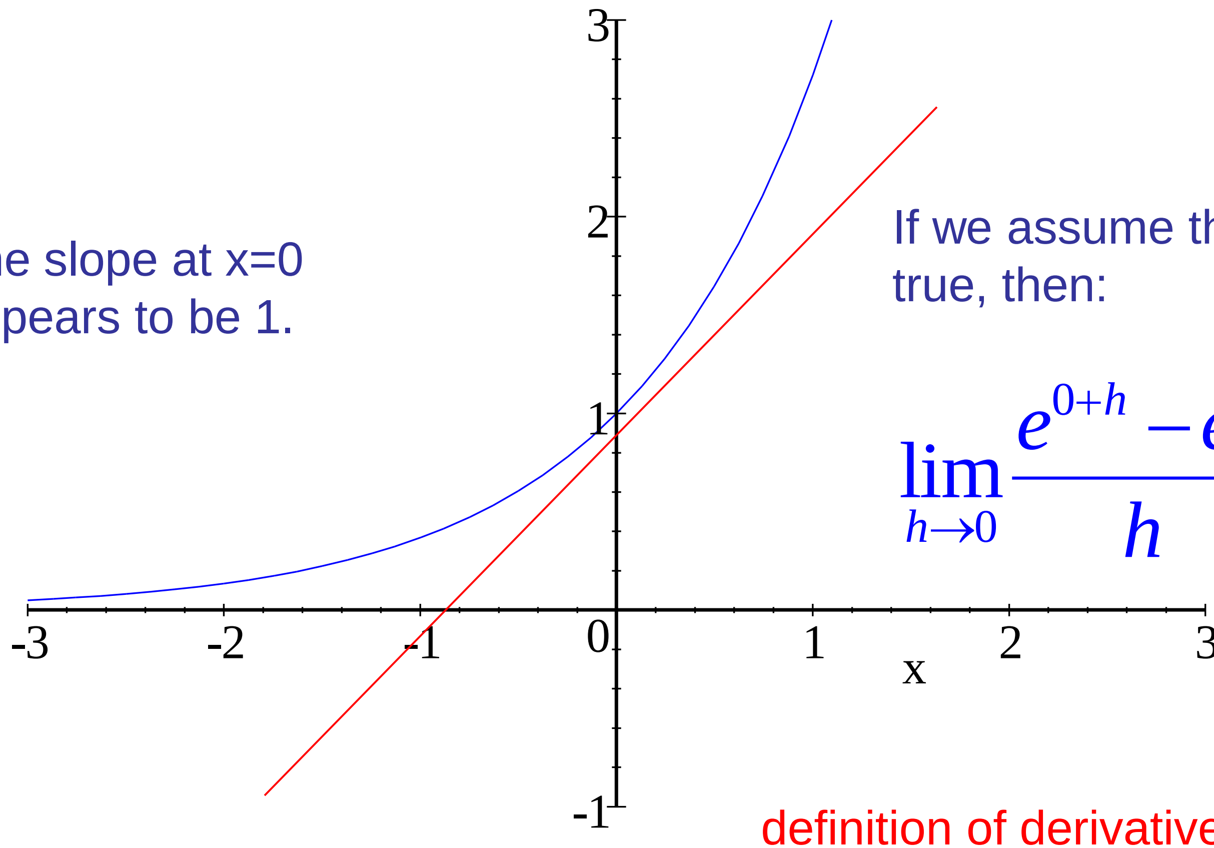
Warm-up

Find $y' = dy/dx$ for $(x\text{-}y)^2 = x + y - 1$

Find $\frac{d^2y}{dx^2}$ where $y^2 - xy = 8$

3.9: Derivatives of Exponential and Logarithmic Functions

Look at the graph of $y = e^x$



The slope at x=0 appears to be 1.

If we assume this to be true, then:

$$\lim_{h \to 0} \frac{e^{0+h} - e^0}{h} = 1$$

definition of derivative

→

Now we attempt to find a general formula for the derivative of $y = e^x$ using the definition.

$$\frac{d}{dx}\left(e^x\right) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} \left(e^x \cdot \frac{e^h - 1}{h} \right)$$

$$= e^x \cdot \lim_{h \to 0} \left(\frac{e^h - 1}{h} \right)$$

This is the slope at x=0, which we have assumed to be 1.

$$= e^x \cdot 1$$

$$= e^x$$

→

The Derivative of e^x

Therefore: The derivative of $f(x) = e^x$ is $f'(x) = e^x$.

Find f’(x)

A) $f(x) = 4e^x – 8x^2 + 7x - 14$

$f'(x) = 4e^x – 16x + 7$

B) $f(x) = x^7 – x^5 + e^3 – x + e^x$

$f'(x) = 7x^6 – 5x^4 + 0 –1 + e^x$

$= 7x^6 – 5x^4 –1 + e^x$

Review: properties of ln

1) $\ln(ab) = \ln a + \ln b$

2) $\ln \frac{a}{b} = \ln a - \ln b$

3) $\ln a^k = k \ln a$

4) $\ln e = 1$

5) $\ln 1 = 0$

The Derivative of *ln x*

Therefore: The derivative of $f(x) = ln\ x$ is $f'(x) = \dfrac{1}{x}$

Find y’ for

A) $y = 10x^3 - 100\ln x$

$$y' = 30x^2 - 100\left(\frac{1}{x}\right) = 30x^2 - \frac{100}{x}$$

B) $y = \ln x^5 + e^x - \ln e^2$

$$y = 5\ln x \;\; + e^x - \ln e^2$$

$$y' = 5\left(\frac{1}{x}\right) + e^x + 0 \;\; = \frac{5}{x} + e^x$$

More formulas

The derivative of $f(x) = b^x$

is $f'(x) = b^x \ln b$

The derivative of $f(x) = \log_b x$

is $f'(x) = \frac{1}{x}\left(\frac{1}{\ln b}\right)$

Find g’(x) for

A) $$g(x) = x^{10} + 10^x$$

$$g'(x) = 10x^9 + 10^x \ln(10)$$

B) $$g(x) = \log_2 x - 6\log_5 x$$

$$g'(x) = \frac{1}{x}\left(\frac{1}{\ln 2}\right) - 6\left(\frac{1}{x}\right)\left(\frac{1}{\ln 5}\right)$$

$$g'(x) = \frac{1}{x}\left(\frac{1}{\ln 2} - \frac{6}{\ln 5}\right)$$