4.1 Antiderivatives and Indefinite Integration

Suppose we have a function F whose derivative is given as $F'(x) = f(x) = x^2$. From your experience with finding derivatives, you might say that $F(x) =$ WHAT???? How can you check your answer?????

Congratulations, you have just found *an* **antiderivative**, F, of f.

Definition

A function F is *an* **antiderivative** of f on an interval I if $F'(x) = f(x) \quad \forall x \in I$.

Notice that F is called AN antiderivative and not THE antiderivative. This is easily understood by looking at the example above.

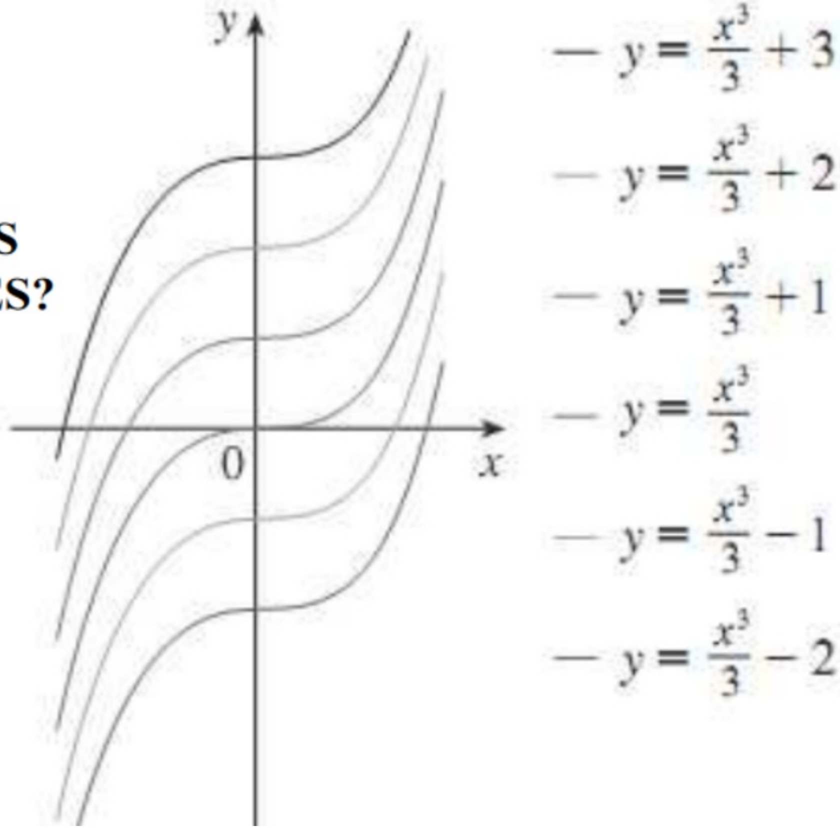
Some antiderivatives of $f(x) = x^2$ are $F(x) = \frac{1}{3}x^3$, $F(x) = \frac{1}{3}x^3 + 3$, $F(x) = \frac{1}{3}x^3 - 2$, and $F(x) = \frac{1}{3}x^3 + \pi$ because in each case, $\frac{d}{dx}\left[F(x)\right] = x^2$.

antiderivative of a function $f(x)$ is

$F(x)+C$, where C is an arbitrary constant.

The graph at right show several members of the family of the antiderivatives of x^2.

WHAT GRAPHICAL CONSEQUENCE DOES THE +*C* HAVE ON THE SOLUTION CURVES?



Find the general antiderivatives of each of the following using you knowledge of how to find derivatives.

(a) $f(x) = 2x$

(b) $f'(x) = x$

(c) $F'(x) = \frac{2}{3}x^{\frac{4}{7}}$

(e) $\frac{dy}{dx} = \cos x$

(d) $g'(x) = \frac{1}{x^2}$

Function	General antiderivative
$cf(x)$	$cF(x)+C$
$f(x)\pm g(x)$	$F(x)\pm G(x)+C$
$x^n,\ n\neq -1$	$\frac{1}{n+1}x^{n+1}+C$
$\frac{1}{x}$	$\ln\lvert x\rvert+C$
e^x	e^x+C
$\cos x$	$\sin x+C$
$\sin x$	$-\cos x+C$
$\sec^2 x$	$\tan x+C$

Function	General antiderivative
$\csc^2 x$	$-\cot x+C$
$\sec x\tan x$	$\sec x+C$
$\csc x\cot x.$	$-\csc x+C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}x+C$
$\frac{1}{1+x^2}$	$\tan^{-1}x+C$
$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1}\lvert x\rvert+C$

Find all functions g such that $g'(x) = 4\sin x + \dfrac{2x^4 - \sqrt{x} + x}{x} - \dfrac{7x\csc^2 x + 1}{x}$.

Definition

A **differential equation** is an equation that has a derivative in it. **Solving a differential equation** involves finding the original function from which the derivative came. The **general solution** involves $+C$. The **particular solution** uses an initial condition to find the specific value of C.

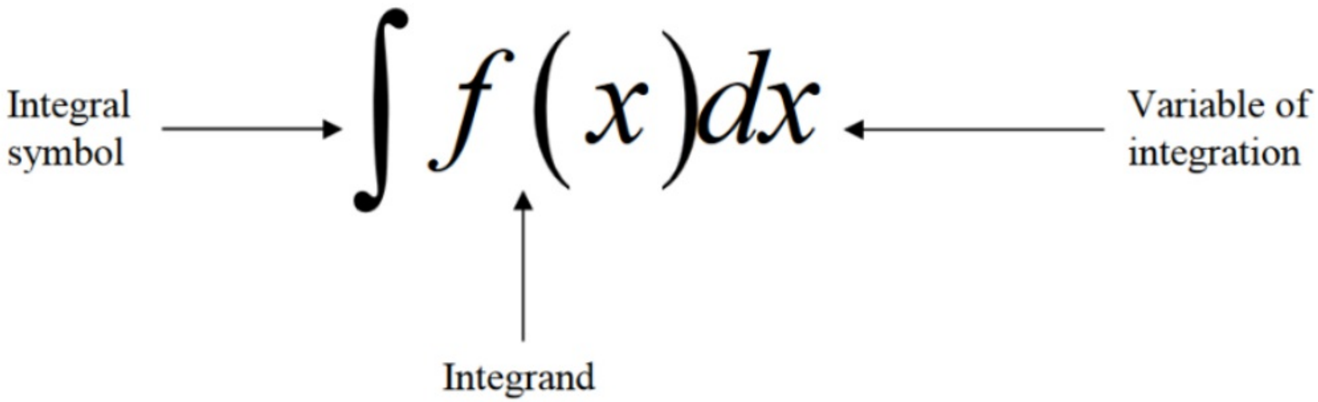
Example 3:

Solve the differential equation $f'(x) = 3x^2 + 1$ if $f(2) = -3$. Find both the general and particular solutions.

Find the particular solution to the following differential equation if $\frac{dy}{dx} = e^{[illegible]} + 20\left(1+x^{[illegible]}\right)^{[illegible]}$ and $y(0) = -2$.

When we are asked to take the derivative of an expression, we have the verb notation

$$\frac{d}{dx}\left[f(x)\right] =$$



Find the particular solution to the following differential equation if $\frac{d^2y}{dx^2} = 12x^2 + 6x - 4$ and

(a) $y'(1) = 3$ and $y(0) = -6$

(b) $y(0) = 4$ and $y(1) = 1$.

(a) $\int \left[\frac{5\sqrt{1-x^2}}{3-3x^2} \right] dx$

(b) $\int \frac{\sin t}{\cos^2 t} dt$

(c) $\int \left(\tan^2 p + 4 \right) dp$

(d) $\int 3\cos^2\left(\frac{m}{2}\right) dm$

(e) $\int z^3 (3-2z)^2 \, dz$

(f) $\int \left[\frac{x^2 - 5x - 14}{x-7} \right] dx$