4.3 The Fundamental Theorem of Calculus

We've learned two different branches of calculus so far: differentiation and integration. Finding slopes of tangent lines and finding areas under curves seem unrelated, but in fact, they are very closely related. It was Isaac Newton's teacher at Cambridge University, a man name **Isaac Barrow** (1630 – 1677), who discovered that these two processes are actually **inverse operations** of each other in much the same way division and multiplication are. It was Newton and Leibniz who exploited this idea and developed the calculus into it current form.

The Theorem Barrow discovered that states this inverse relation between differentiation and integration is called **The Fundamental Theorem of Calculus**.

The Fundamental Theorem of Calculus, Part 1 (FTOC1)

If f is continuous on $[a,b]$ and $F(x)$ is an antiderivative of f, then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

This integral gives us the NET change!!!

Example 1:

Evaluate the integrals using the FTOC1.

(a) $\int_1^3 \left(e^x - 3\right) dx$ (b) $\int_0^{1/2} \frac{2}{\sqrt{1-x^2}} dx$ (c) $\int_1^{e^2} x^{-1} dx$

Example 2:

Evaluate each definite integral using **any** method. Verify on the calculator.

(a) $\int_{1}^{2}\left(x^2-3\right)dx$ (b) $\int_{1}^{4}3\sqrt{x}dx$ (c) $\int_{0}^{\pi/4}\sec^2 x dx$ (d) $\int_{0}^{2}\left|2x-1\right|dx$

Example 3:

Find the exact value of $\int_{1/2}^{1/e}\left(10x^4-2\left(1-x^2\right)^{-1/2}-\frac{5}{x}\right)dx$

area is always POSITIVE, and WE are responsible for making negative regions positive!!

In such cases, we must CLEARLY IDENTIFY THE INDICATED REGION!

Example 4:

Find the area bounded by the parabola $y = x^2 - 1$ and $y = 0$ from $x = 0$ to $x = 3$

(a) Without a calculator

(b) With a calculator

Example 5:

Find the area of the region bounded by the curves $y=0$, $y=\frac{2}{x}-1$, $x=1$ and $x=e$

(a) Without a calculator

(b) With a calculator

Example 6:

Without a calculator, find the area of the region bounded by the x-axis and the function $y = \dfrac{-20}{x^2+1}$ on the interval $-1 \le x \le 1$. Use the symmetry of the function to help you evaluate the integral.