4.3b FTOC Part 2 and MVT (for integrals)

The second part of the theorem deals with integral equations of the form

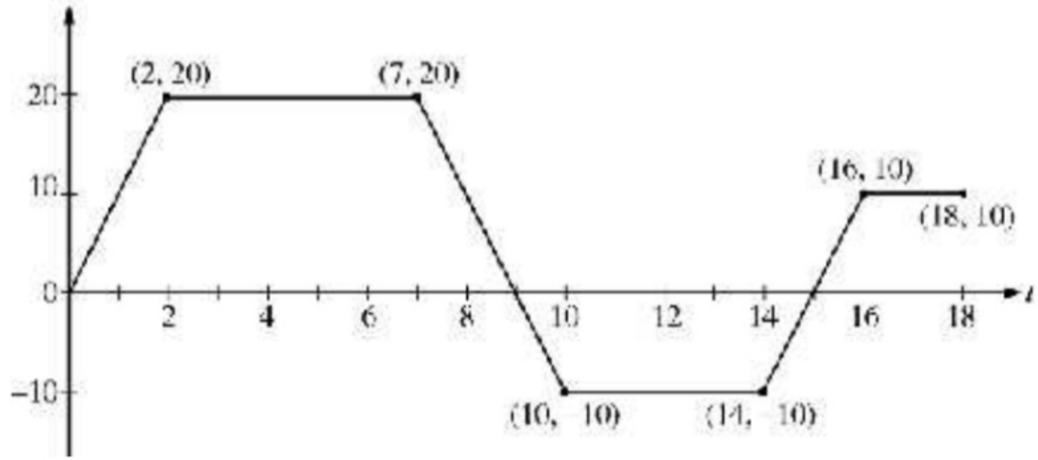
$$F(x)=\int_a^x f(t)\,dt$$

where *f* is a continuous function on $[a,b]$, and *x* varies between *a* and *b*. Notice that this integral equation is a function of *x*, which appears as the upper limit of integration. If $f(t)$ happens to be positive, and we let $x\in(a,b]$, then we can define $F(x)$ as the area under the curve from *a* to *x*.

Example 7:

The graph of $f(t)$ is given. Let $F(x)=\int_{2}^{x} f(t)dt$. Use the areas of the regions to find the following:

(a) $F(2)$



(b) $F(9)$

(c) $F(15)$

(d) $F(18)$

(e) $F(0)$

Example 8:

If $F(x)=\int_3^x \left(t^2+t+1\right)dt$, find a simplified, expanded version of $F(x)$ by evaluating the definite integral.

Once you find $F(x)$, find its derivative, $F'(x)$. What do you notice?

The Fundamental Theorem of Calculus Part 2 (FTOC2)—special case

If *f* is a continuous function on $[a,b]$, then the function *F* defined by

$$F(x)=\int_a^x f(t)\,dt,\ a\le x\le b$$

is continuous on $[a,b]$ and differentiable on (a,b). Additionally, $F'(x)=f(x)$. We can also say that

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right]=f(x)$$

Example 9:

Evaluate (a) $\frac{d}{dx}\left[\int_{1}^{x} \sec t \; dt\right] =$ (b) $\frac{d}{dx}\left[\int_{5}^{x} \sqrt{p^3} \sin p \; dp\right] =$ (c) $\frac{d}{dx}\left[\int_{\pi}^{x} \frac{e^k \sqrt{k+k^2}}{\ln k} \; dk\right] =$

The FTOC2, most general form:

$$\frac{d}{dx}\left[\int_{h(x)}^{g(x)} f(t)dt\right] = f\left(g(x)\right)\cdot g'(x) - f\left(h(x)\right)\cdot h'(x)$$

Example 10:

Evaluate the following using the FTOC2, then verify by doing in the Looooooong way.

(a) $\frac{d}{dx}\left[\int_{1}^{2x^3} \sec^2 t \, dt\right]$

(b) $\frac{d}{dx}\left[\int_{e^x}^{7} \left(t^2 + 5t\right)dt\right]$

Example 11:

If $F(x) = \int_{2+\sin 2x}^{3^x} \ln t \, dt$, find $F'(x)$

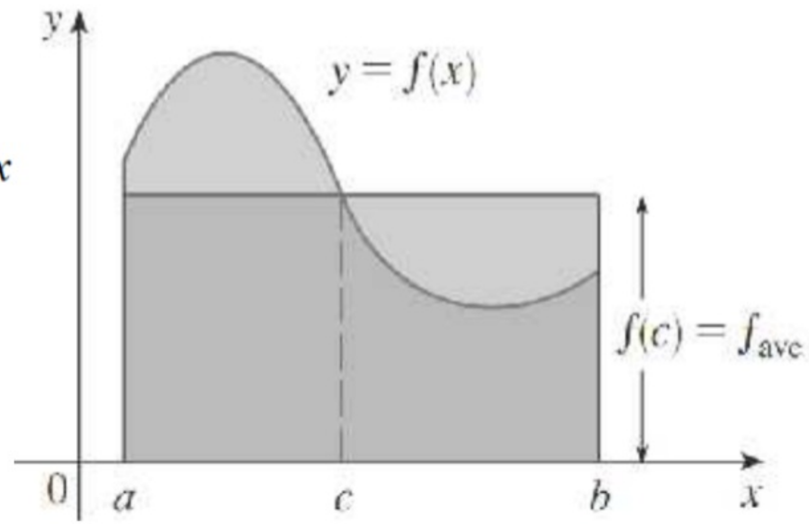
The Mean Value Theorem (for Integrals)

If f is continuous on the closed interval $[a,b]$, then there exists a number $x=c$ in the CLOSED interval $[a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$

Where $f(c)$ is called the **average value** of the function f on the interval $[a,b]$. The above equation above can be explicitly solved for $f(c)$.

$$f(c)=\frac{\int_a^b f(x)\,dx}{b-a} \quad \text{or} \quad f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$$



Example 12:

(Calculator) In New Braunfels, the temperature (in $^\circ F$) t hours after 9 a.m. was modeled by the function $T(t) = 50 + 14\sin\frac{\pi t}{12}$. (a) Find the average temperature during the 12-hour period from 9 a.m. to 9 p.m. **<u>Show your integral set up</u>**! (b) find the value(s) of t at which the MVT guarantees the temperature in N.B. was the average temperature.

Example 13:

Find the value of c guaranteed by the MVT for integrals for $f(x) = 1 + x^2$ on $[-1, 2]$. Interpret the result graphically.