4.3c MVT for Integrals

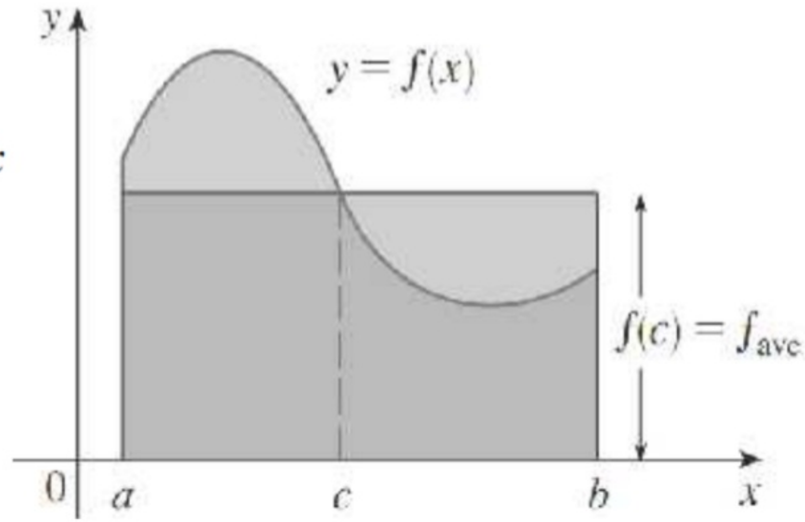
The Mean Value Theorem (for Integrals)

If *f* is continuous on the closed interval $[a,b]$, then there exists a number $x=c$ in the CLOSED interval $[a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$

Where $f(c)$ is called the **average value** of the function *f* on the interval $[a,b]$. The above equation above can be explicitly solved for $f(c)$.

$$f(c)=\frac{\int_a^b f(x)\,dx}{b-a} \quad \text{or} \quad f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$$



Example 12:

(Calculator) In New Braunfels, the temperature (in $^\circ F$) t hours after 9 a.m. was modeled by the function $T(t) = 50 + 14\sin\frac{\pi t}{12}$. (a) Find the average temperature during the 12-hour period from 9 a.m. to 9 p.m. **<u>Show your integral set up</u>**! (b) find the value(s) of t at which the MVT guarantees the temperature in N.B. was the average temperature.

Example 13:

Find the value of c guaranteed by the MVT for integrals for $f(x) = 1 + x^2$ on $[-1, 2]$. Interpret the result graphically.

Example 14:

Show that the average rate of change of a car's position over a time interval $[t_1, t_2]$ is the same as the average value of its velocity function over the same interval.

Example 15:
The table below gives values for a continuous function. Using the values given, find the arithmetic mean of $f(x)$. Using a trapezoidal approximation using 6 subintervals, estimate the average value of f on $[20, 50]$. For this continuous function, is it possible to determine which is more accurate? Is one method better practice than the other?

x	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

Example 16:

Find the number(s) b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

Example 17:
Solve the following conditional equations for the indicated variable.

(a) $\int_{-5}^{x} \left(t^3 - 7t\right) dt = 0$, for x

(b) $\int_{-3}^{k} \left(x^2 - 4\right) dx = 0$, for k

IMPORTANT Example 18:

If $f'(x) = 2x^2 - 2$ and $f(0) = 5$, find $f(2)$ by (a) finding the particular solution to the differential equation, then evaluating the solution at $x = 2$, and then by (b) using a definite integral.

Important Idea of Accumulation**********************************(* means VERY IMPORTANT)

What I have now = What I started with + What I've accumulated since I started

This can be expressed mathematically as

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$

Example 19:

If $f'(x) = 4\sin^2(2x)$ and $f(2) = -2$, using your calculator, (a) through (c) only, find the following. Be sure to show your INTEGRAL SET UP for parts (a) through (c).

(a) $f(3)$ (b) $f(5)$ (c) $f(-2)$ (d) an integral equation for $f(x)$