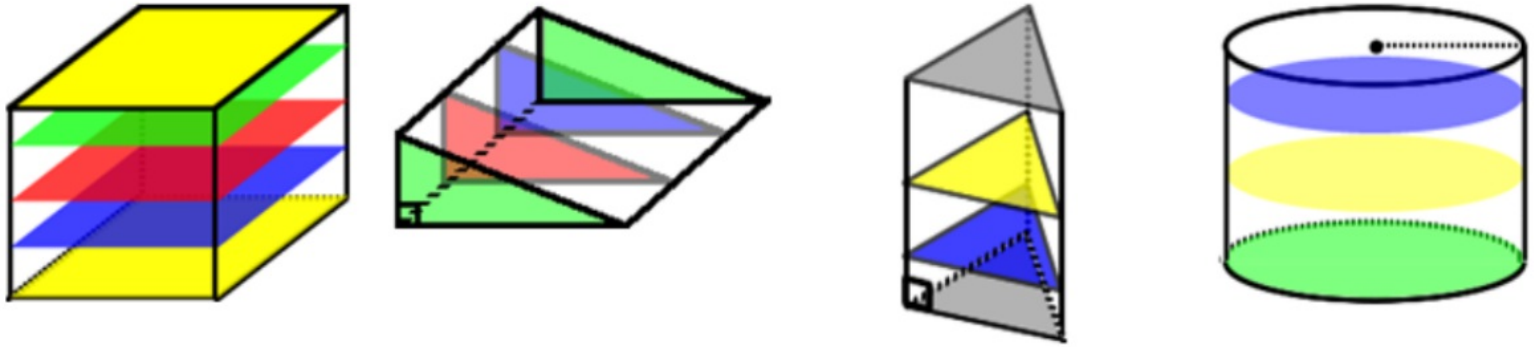
Warm-up

491. Find the equation of the tangent line to the curve $y = \sqrt{x^2 - 3}$ at the point $(2, 1)$.

493. Find the equation of the tangent line to the curve $y = \sqrt{3x - 1}$ that is perpendicular to the line $3y + 2x = 3$.

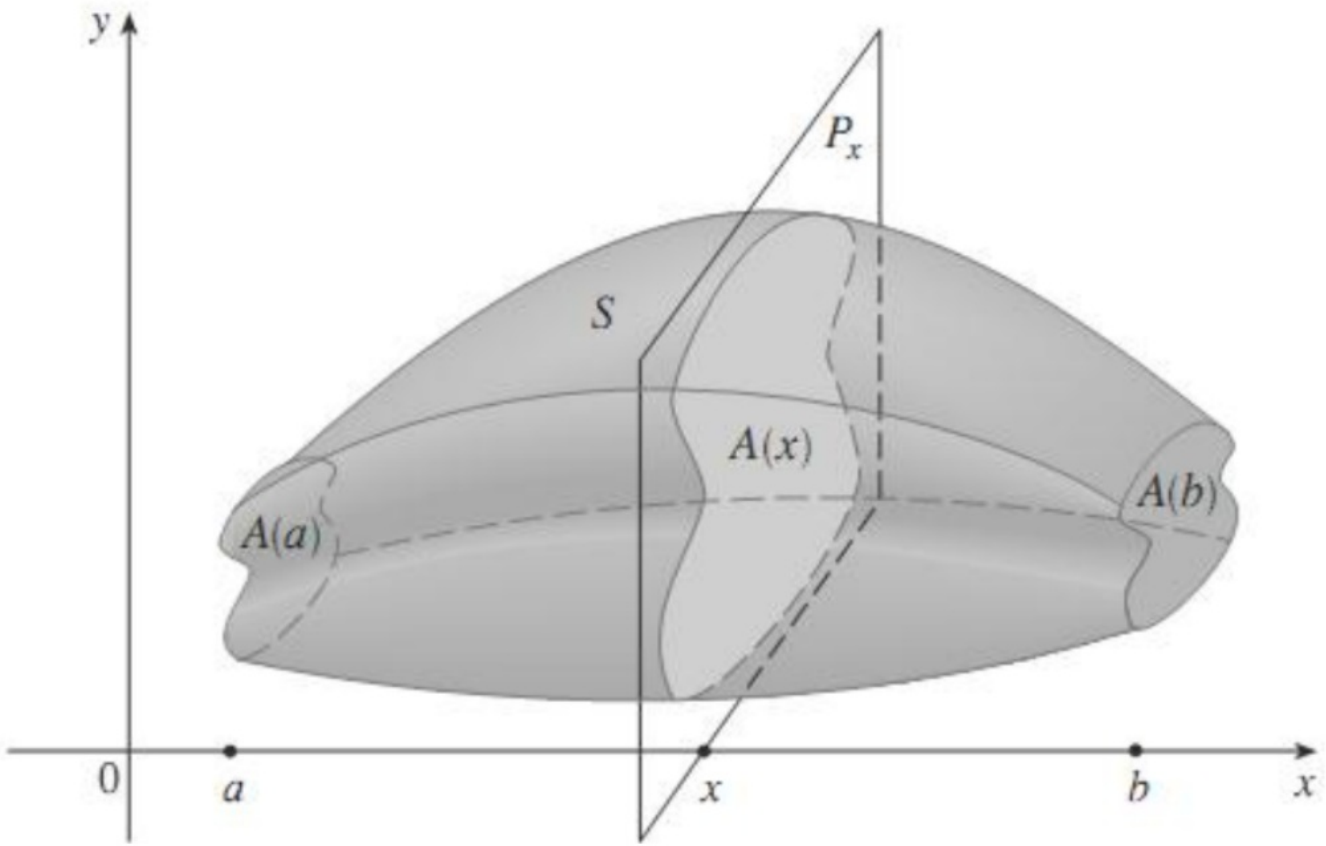
5.2 Volumes Using Disk Method

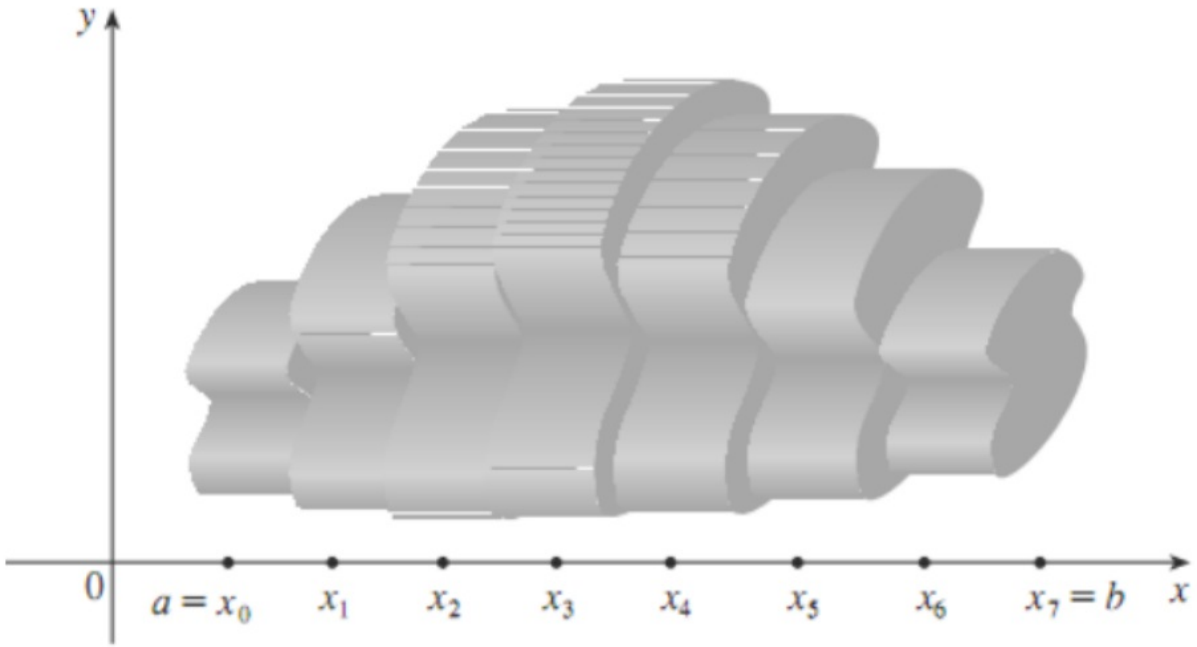
$$V = A \cdot h$$



For the shapes above, the area of the face is the same at every point sliced parallel to the face. This is not always the case, and this is where calculus comes in.

Imagine slicing a loaf of bread (mathematically). It might look like this





We can approximate the volume of the entire loaf by finding the volumes of each slice $V(x_i) = A(x_i) \cdot \Delta x$ and adding them up

Disc Method for Volumes of Solids of Rotation

When the volume of solid is obtained by rotating a region **perpenDISCular** to the axis of rotation and the cross-sections are discs or circles, the volume of the solid is given by

$$V = \pi \int_{}^{b} R(x)^2 \, dx$$

Where $R(x)$ is the radius of rotation as a function of x.

Example 1

Find the volume of the solid formed by rotating the region bounded by the x-axis, $y = \sqrt{x}$, and $x = 1$ around the x-axis.

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Example 2

Find the volume of the solid formed by rotating the region bounded by the $y = 1$, $y = \sqrt{x}$, and $x = 0$ around the line $y = 1$.

Example 3

Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.

Example 4

What if we were to take the region from the previous example and rotate it around the x-axis instead of the y-axis? What would the shape look like? What would a perpendicular slice look like? Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the x-axis.

Practice

1. Find the volume of the solid formed when the region bounded by the curves $y = x^3 + 1$, $x = 1$ and $y = 0$ is rotated about the x-axis.

2. Find the volume of the solid of revolution obtained by revolving the region bounded by $y = 1/x$ and the lines $x = \pi/8$ and $x = \pi/2$ around the x axis

3. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^2 - 4x + 5$ and $y = 5 - x$ about the line $y = -1$.

4. Find the volume of the solid generated by revolving about the line $x = -1$, the region bounded by the curves $y = -x^2 + 4x - 3$ and $x = 6$