14. If $g(t) = \dfrac{\ln t}{e^t}$, then $g'(t) =$

A) $\dfrac{1 - \ln t}{e^t}$

B) $\dfrac{1 - t \ln t}{e^t}$

C) $\dfrac{t \ln t - 1}{te^t}$

D) $\dfrac{1 - t \ln t}{te^t}$

E) $\dfrac{1 - e^t \ln t}{e^{2t}}$

15. If $H(x) = x^3 - x^2 + \dfrac{1}{x}$, which of the following is $H''(2)$?

A) $\frac{31}{4}$

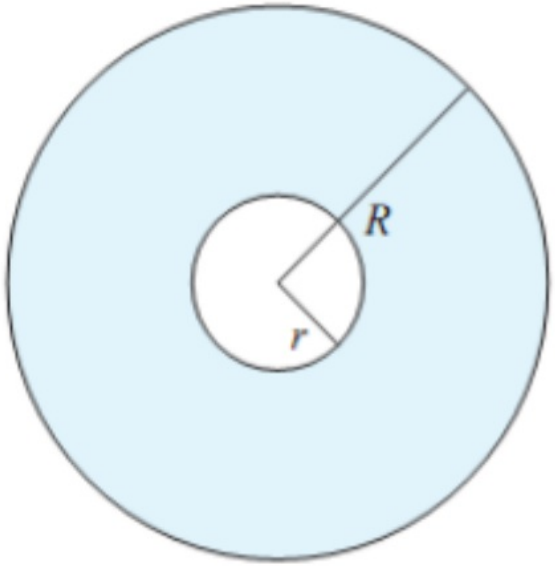
B) $\frac{39}{4}$

C) $\frac{79}{8}$

D) $\frac{81}{8}$

E) $\frac{41}{4}$

Sometimes, our cross sections are circles but have a void or hole in them.



In this case, the area of the face of the cross section will be

$$A(x) = \pi R(x)^2 - \pi r(x)^2 = \pi\left(R(x)^2 - r(x)^2\right)$$

Washer Method for Volumes of Solids of Rotation/Revolution

When the volume of solid is obtained by rotating a region **perpenWASHular** to the axis of rotation and the cross-sections are washers, the volume of the solid is given by

$$V = \pi \int_a^b \left[R(x)^2 - r(x)^2 \right] dx$$

Where $R(x)$ is the larger, outer radius of rotation and $r(x)$ is the smaller, inner radius rotation.

Example 5

The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.

Important things to consider when using the Washer method:

- Draw a picture, draw a picture, draw a picture, . . . You must identify the region 1st!
- Like the Disc method, the cross-sections (slices/representative rectangles) must be PERPENDICULAR to the axis of rotation/revolution
- Before writing an equation for R and r, draw them on your diagram. If you can draw them, you can write them.
- When writing an equation for R and r, it will still involve $TOP - BOTTOM$ (vertical slice) or $RIGHT - LEFT$ (horizontal slice). **One of these in each case will be the axis of rotation itself.**
- **DON'T FORGET TO SQUARE EACH RADIUS BEFORE SUBTRACTING THEM.** The most common error is to integrate as $V = \pi \int_a^b \left[\left(R(x) - r(x) \right)^2 \right] dx$. This is **WRONG**. Keep telling yourself that you're subtracting two separate volumes: $\pi R^2 dx - \pi r^r dx$. The π and dx are simply factored out.

Example 6

Find the volume of the solid formed when the R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the following axes:

(a) the x-axis.

(b) the line $y = 2$

(c) the line $y = -5$

(d) the y-axis

(e) the line $x = -1$

(f) the line $x = 17$