Sometimes, an integral can be doubly improper.

If $\int_{-\infty}^{c} f(x)dx$ and $\int_{c}^{\infty} f(x)dx$ are both convergent, then $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_{c}^{\infty} f(x)dx$, where c is any number. Symmetry can also be used to circumvent the doubleness of the impropriety. Note as well that this requires BOTH of the integrals to be convergent in order for this integral to also be convergent. If either of the two integrals is divergent then so is this integral.

Example 9:

(a) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2} =$

(b) $\int_{-\infty}^{\infty} xe^{-x^2}\,dx =$

Example 10:

(Calculator permitted) Find the area of the region bounded by the graph of $y=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ and the x-axis.

Comparison Test for Convergence or Divergence

Let f and g be continuous on $[a,\infty)$ with $0 \le f(x) \le g(x)$ for all $x \ge a$, then

- If $\int_a^\infty g(x)\,dx$ converges, then $\int_a^\infty f(x)\,dx$ converges too!
- If $\int_a^\infty f(x)\,dx$ diverges, then $\int_a^\infty g(x)\,dx$ diverges too!

Example 11:

(a) If $0 \le e^{-x^2} \le e^{-x}$ for all $x \ge 1$, determine if $\int_1^\infty e^{-x^2}\,dx$ converges or diverges.

(b) Determine if $\int_{\pi}^{\infty} \frac{2+\cos x}{x} dx$ converges or diverges.

(c) Determine if $\int_{4}^{\infty} \frac{2}{x+e^{x}} dx$ converges or diverges.

(d) Determine if $\int_{4}^{\infty} \frac{2}{x - e^{-x}}\, dx$ converges or diverges.