6.3 Euler's Method

FIRST BC TOPIC!!!!

Euler's method basically involves "walking out along a tightrope" from an initial point along mutliple tangent lines. Instead of walking along the same line the whole time (as in a tangent line approximation), we change tangent lines with each step (of length Δx).

- We first must designate the number of equal steps we would like to take. Call this number n.
- Next, if $x = a$ is our initial x-value, and $x = b$ is our desired x-value to find our approximation at, we calculate Δx the conventional way:

$$\Delta x = \frac{b-a}{n}$$

- Recall slope: $m = \frac{\Delta y}{\Delta x}$, solving for Δy, we get $\Delta y = m(\Delta x)$
- We can now proceed. The following chart will make things easier.

(note: the first x and y used are the initial condition. $\frac{dy}{dx}$ will be given)

MEMORIZE THIS CHART. MEMORIZE THIS CHART.

x	y	$m = \left.\frac{dy}{dx}\right\|_{(x,y)}$	$\Delta y = m(\Delta x)$	$y_{new} = y + \Delta y$
a	$y(a)$			

Example 1:

Given the differential equation $\frac{dy}{dx} = x - 2$ and $y(0) = 5$.

(a) Find an approximation for $y(0.8)$ by using Euler's method with two equal steps. Sketch your solution.

(b)Solve the differential equation $\frac{dy}{dx} = x - 2$ with the initial condition $y(0) = 5$, and use your solution to find $y(0.8)$.

Example 2:

If $\frac{dy}{dx} = 2x - y$ and if $y = 3$ when $x = 2$, use Euler's method with five equal steps to approximate y when $x = 1.5$.

Example 3:

Assume that f and f' have the values given in the table. Use Euler's method with two equal steps to approximate the value of $f(2.6)$.

x	3	2.8	2.6
$f'(x)$	0.4	0.7	0.9
$f(x)$	2		

Example 4:

x	$g'(x)$
–1.0	2
–0.5	4
0.0	3
0.5	1
1.0	0
1.5	–3
2.0	–6

The table at left gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) –6.5 (B) –1.5 (C) 1.5 (D) 2.5 (E) 3