6.4 Integration by Parts

$$\int x^2 \sin x dx$$

We have two factors. We know how to differentiate one (or both) of them and we know how to integrate the other one (or both) of them.

When you see this pattern, it's time for integration by parts.

Example 1:
Evaluate $\int x \cos x dx$

Why the heck does that work?

Remember the product rule of differentiation (don't answer that if you don't).

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Rearranging and integrating both sides, we get:

$$\int f'(x)g(x)dx + \int f(x)g'(x)dx = \int \frac{d}{dx}\left[f(x)g(x)\right]dx$$

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

If we, for the sake of simplicity, let $u = f(x)$ and $v = g(x)$, we get

$$\boxed{\int udv = uv - \int vdu}$$

Example 2:

Evaluate $\int 2xe^{4x}dx$

Example 3:

Evaluate $\int x \ln x dx$

Example 4:

Evaluate $\int \ln x dx$

Example 5:

Evaluate $\int \arctan x dx$

Example 6:

Evaluate $\int x \arctan x dx$

Example 7:

Evaluate $\int \theta \sec^{-1} \theta d\theta$

Example 8:

Evaluate $\int x^2 \sin x dx$. In the movie, Tito was using the "backwards Zorro" method, and chose $u = \sin x$ and $dv = x^2 dx$. Use Tito's method and feel his frustration. Learn from it.

The "Tic-Tac-Toe" a.k.a. the Tabular Method

In general, we like to choose our polynomial term as u. The exception would be if there is a log or inverse trig factor. When our polynomial is degree two or higher, we can abandon the "repeated backwards Zorro" method for the Tabular method. It's as easy as Tic-Tac-Toe.

Example 9:

Evaluate $\int x^2 \sin x dx$ using the tabular method.

Example 10:

Evaluate $\int t^4 e^{2t}\,dt$

Example 11:

Solve $\frac{dz}{dx} = x^3 \ln x$, if $z = 5$ when $x = 1$

Example 12:

If $f(x) = x^3 \cos 2x$, find the average value of f on the interval $\left[0, \frac{\pi}{2}\right]$. Verify on your calculator.

Example 13:

Evaluate $\int \frac{e^x}{\sec x} dx$