Name______________________________________ Date______________________ Period_____

Worksheet 7.1—Intro to Parametric & Vector Calculus
Show all work. No calculator unless explicitly stated.

Short Answer

1. If $x = t^2 - 1$ and $y = e^{t^3}$, find $\frac{dy}{dx}$.

2. If a particle moves in the $xy-$plane so that at any time $t > 0$, its position vector is $\left\langle \ln\left(t^2 + 5t\right), 3t^2 \right\rangle$, find its velocity vector at time $t = 2$.

3. A particle moves in the $xy-$plane so that at any time t, its coordinates are given by $x = t^5 - 1$, $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.

4. If a particle moves in the $xy-$plane so that at time *t*, its position vector is $\left\langle \sin\left(3t-\frac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t=\frac{\pi}{2}$.

5. A particle moves on the curve $y=\ln x$ so that its *x*-component has velocity $x'(t)=t+1$ for $t \geq 0$. At time $t=0$, the particle is at the point $(1,0)$. Find the position of the particle at time $t=1$.

6. A particle moves in the $xy-$plane in such a way that its velocity vector is $\langle 1+t, t^3 \rangle$. If the position vector at $t=0$ is $\langle 5,0 \rangle$, find the position of the particle at $t=2$.

7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

8. The position of a particle moving in the $xy-$plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?

9. A curve C is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write an equation of the line tangent to the graph of C at the point $(8, -4)$.

10. (Calculator Permitted) A particle moves in the xy – plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$. Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

Free Response:

11. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at time $t = 5$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

(c) Find $\frac{dy}{dx}$ as a function of *x*.

12. Point $P(x, y)$ moves in the $xy-$plane in such away that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$.

(a) Find the coordinates of *P* in terms of *t* when $t = 1$, $x = \ln 2$, and $y = 0$.

(b) Write an equation expressing *y* in terms of *x*.

(c) Find the average rate of change of *y* with respect to *x* as *t* varies from 0 to 4.

(d) Find the instantaneous rate of change of *y* with respect to *x* when $t = 1$.

13. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.

(a) Find $\frac{dy}{dx}$ as a function of *t*.

(b) Find an equation of the tangent line at the point where $t = \frac{\pi}{4}$.

(c) (Calculator Permitted) The curve C intersects the *y*-axis twice. Approximate the length of the curve between the two *y*-intercepts.

Multiple Choice:

14. A parametric curve is defined by $x = \sin t$ and $y = \csc t$ for $0 < t < \frac{\pi}{2}$. This curve is

(A) increasing & concave up (B) increasing & concave down (C) decreasing & concave up
(D) decreasing & concave down (E) decreasing with a point of inflection

15. The parametric curve defined by $x = \ln t$, $y = t$ for $t > 0$ is identical to the graph of the function

(A) $y = \ln x$ for all real x (B) $y = \ln x$ for $x > 0$ (C) $y = e^x$ for all real x
(D) $y = e^x$ for $x > 0$ (E) $y = \ln\left(e^x\right)$ for $x > 0$

16. The position of a particle in the $xy-$plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$ for all $t \geq 0$. The acceleration vector of the particle is

(A) $\left(2t, \frac{2}{2t+3}\right)$ (B) $\left(2t, -\frac{4}{(2t+3)^2}\right)$ (C) $\left(2, \frac{4}{(2t+3)^2}\right)$
(D) $\left(2, \frac{2}{(2t+3)^2}\right)$ (E) $\left(2, -\frac{4}{(2t+3)^2}\right)$