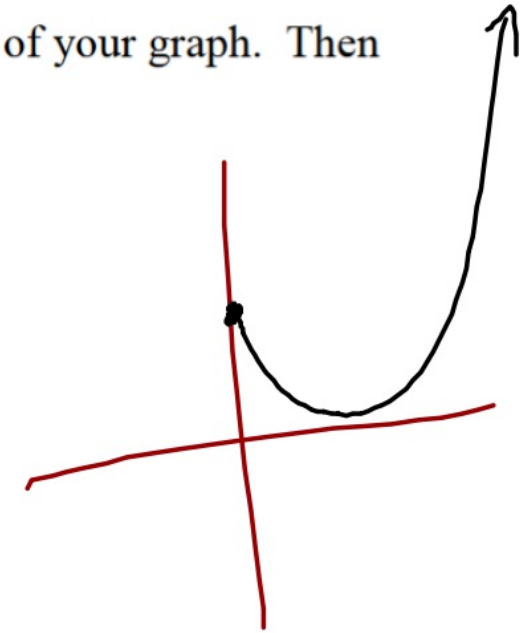
7.1 Intro to Parametric and Vector Calculus

In Algebra, equations are graphed in two variables, x and y. Now we will graph equations with x, y, and t, or with x, y, and θ, where x and y are expressed independently in terms of t or θ. The third variable, t or θ is called the parameter, and the separate equation are called parametric equations.

Example 1:
Without a calculator, make a table, and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Verify on your calculator.

$$x = t^2 - 4 \text{ and } y = \frac{t}{2}, \; -2 \le t \le 3$$



What do you notice about the graphs of $x = 4(2\sin t + 1)^2 - 4$ and $y = 2\sin t + 1,\ -1.571 \le t \le 0.253$

While rectangular equations on restricted intervals show the _____, parametric equations show the _____, _____, and _____.

Do the same for $x = \dfrac{1}{\sqrt{t+1}}$, $y = \dfrac{t}{t+1}$

Do the same for $x = 2 + 3\cos t$, $y = -1 + 2\sin t$

Parametric Equations & Formulas for Calculus

If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of C at the point (x, y) is given by $\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}}$ where $\dfrac{dx}{dt} \neq 0$, and the second derivative is given by

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] \cdot \frac{dt}{dx} = \frac{\dfrac{d}{dt}\left[\dfrac{dy}{dx}\right]}{\dfrac{dx}{dt}}.$$

Without a calculator, given $x = 2\sqrt{t}$, $y = 3t^2 - 2t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and evaluate at $t = 1$.

Without a calculator, given $x = 4\cos t$, $y = 3\sin t$, write an equation of the tangent line to the curve at the point where $t = \frac{3\pi}{4}$.

Example 6:

Without a calculator, find all points of horizontal and vertical tangency given $x = t^2 + t$, $y = t^2 - 3t + 5$.