$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$ is the length of the arc from $t = a$ to $t = b$

Example 7:

Without a calculator, find the arc length of the given curve if $x = t^2$, $y = 4t^3 - 1$, $0 \le t \le 1$.

Horizontal and Vertical Velocity Component VECTORS

- $x'(t) = \frac{dx}{dt}$ is the rate at which the x-coordinate is changing with respect to t or the velocity of a particle in the horizontal direction.

- $y'(t) = \frac{dy}{dt}$ is the rate at which the y-coordinate is changing with respect to t or the velocity of a particle in the vertical direction.

- $\vec{s} = \langle x(t), y(t) \rangle = (x(t), y(t))$ is the position at any time t.

- $\vec{v} = \langle x'(t), y'(t) \rangle = (x'(t), y'(t))$ is the velocity vector at any time t.

- $\vec{a} = \langle x''(t), y''(t) \rangle = (x''(t), y''(t))$ is the acceleration vector at any time t.

*note: the vectors may or may not be contained within the chevrons $\langle\ \rangle$.

- $\frac{dy}{dx}$ is the rate of change of y with respect to x or the slope of the tangent line to the curve or the slope of the position vector.

- $\frac{d^2y}{dx^2}$ is the rate of change of the slope of the curve with respect to x.

- $|\vec{v}(t)| = \|\vec{v}(t)\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ is the **speed of a particle** or the **magnitude** or the **length** or the **norm** of the velocity vector.

- $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ is the **length of the arc** from $t = a$ to $t = b$ or the **distance traveled** by a particle from $t = a$ to $t = b$.

*Remember that $\int_a^b |v(t)|\, dt$ is the total distance traveled, whether it be along a straight line or curve.

Example 8:

(No Calculator) A particle moves in the $xy-$plane so that at any time t, $t \geq 0$, the position of the particle is given by $x(t) = t^3 + 4t^2$, $y(t) = t^4 - t^3$.

(a) Find the velocity vector at $t = 1$, (b) the speed of the particle at $t = 1$, and (c) the acceleration vector at $t = 1$.

Example 9:

(No Calculator) A particle moves in the $xy-$plane so that $x=\sqrt{3}-4\cos t$ and $y=1-2\sin t$, where $0\le t\le 2\pi$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(b) The path of the particle intersects the x-axis twice. Write an expression that represents the distance traveled by the particle between the two x-intercepts. Do not evaluate.