Warm Up

State the pattern for each step.

- 3, 6, 12, 24, 48, 96,… ×2 ×2 ×2 — ×2 or ·2
- 81, 27, 9, 3, 1, ⅓,… ·$\frac{1}{3}$ ·$\frac{1}{3}$ ·$\frac{1}{3}$ ·$\frac{1}{3}$ — ·$\frac{1}{3}$ or ×$\frac{1}{3}$
- -2, 4, -8, 16, -32, 64, -128 ·-2 ·-2 — ·-2 or $\times^{-2}$

8.2

Geometric Sequences

Geometric Sequences

A geometric sequence is defined as a sequence in which there is a **common ratio** between consecutive terms.

Common Ratio = 2

$$5, 10, 20, 40, 80, 160, 320, \ldots$$

Is the given sequence geometric? If so, identify the common ratio.

5, 15, 45, 135, … ×3 ×3 yes, $r = 3$

15, 30, 45, 60, … No

6, -24, 96, -384, … yes $r = -4$

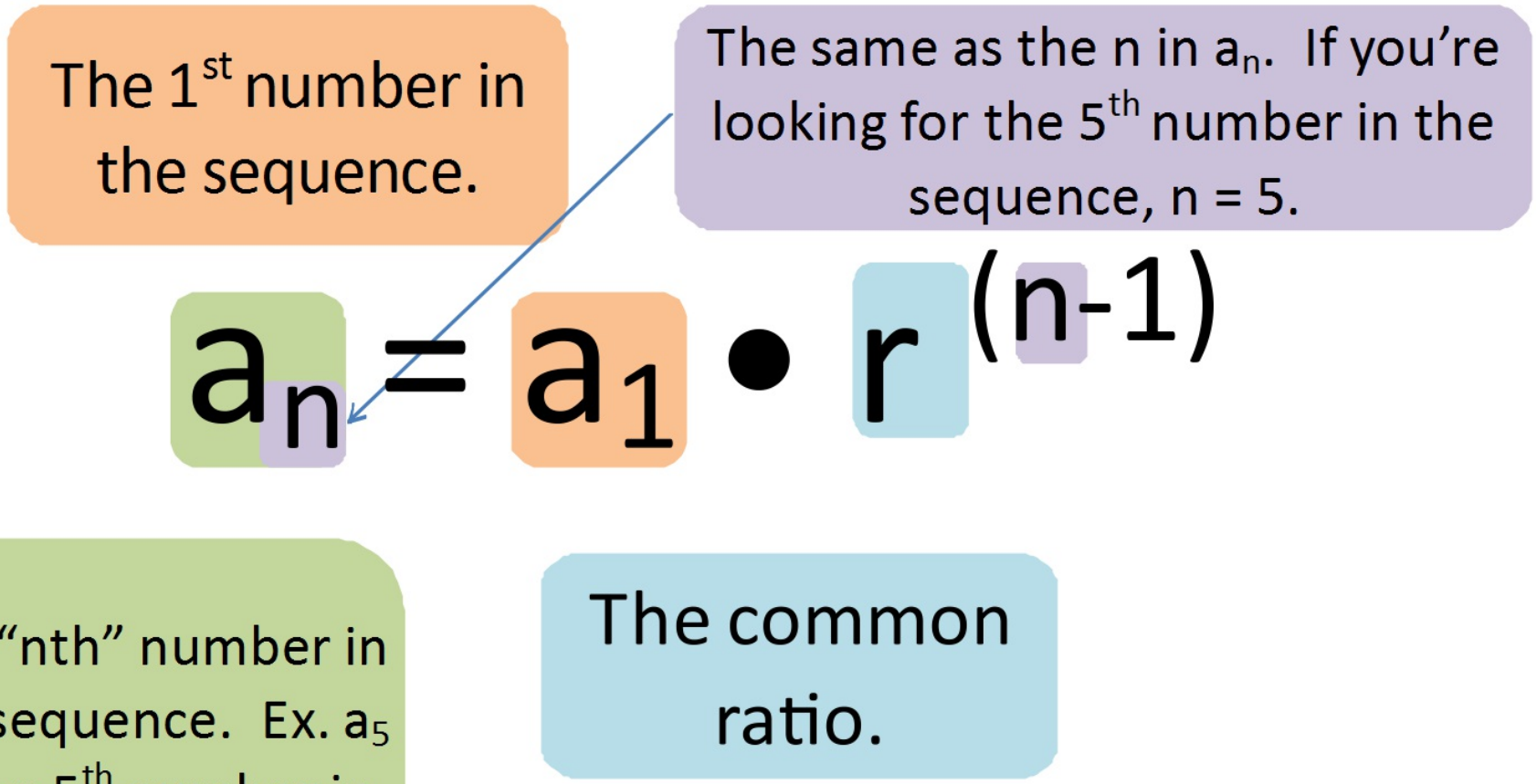
8, 20, 32, 44, … No

1, 2, 4, 8, … ×2 ×2 ×2 yes $r = 2$

7, 0.7, 0.07, 0.007, … yes $r = \frac{1}{10}$ or .1

10, 4, 1.6, 0.64, … yes $r = \frac{1}{2.5}$ or .4 or $\frac{4}{10}$ or $\frac{2}{5}$

Geometric Sequence Formula



Example 1:

$$a_n = a_1 \bullet r^{(n-1)}$$

Given the sequence 4, 28, 196, 1372, 9604,…, find the 7^{th} term.

×7 ×7 ×7

$a_1 = 4$

$r = 7$

$n = 7$

$a_n = ?$

$a_7 = 4 \cdot 7^{(7-1)}$

$a_7 = 4 \cdot 7^6$

$a_7 = 470596$

Pemdas

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Example 2:

$a_n = a_1 \bullet r^{(n-1)}$

$-3 \quad -3$

$6/-2 = -3$

Given the sequence -2, 6, -18, 54, -162,…, find the 17^{th} term.

$a_1 = -2$

$r = -3$

$n = 17$

$a_{17} = -2 \cdot (-3)^{(17-1)}$

$a_{17} = -2 \cdot (-3)^{16}$

$a_{17} = -86093443$

Example 3:

$a_n = a_1 \bullet r^{(n-1)}$

Given the sequence 100, 83, 68.89, 57.1787,…, find the 9th term.

$a_1 = 100$

$r = 83/100 = .83$

$n = 9$

$a_9 = 100 \cdot (.83)^{(9-1)}$

$a_9 = 22.52292232$

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Example 4:

$a_n = a_1 \bullet r^{(n-1)}$

Given the sequence 1, 5, 25, 125, 625, 3125,…, find the term number that is 9,765,625.

$a_1 = 1$

$r = 5$

$n = ? = 11$

$a_n = 9765625$

$\log_5 9765625 = n-1$

$\frac{9765625}{1} = \frac{1 \cdot 5^{(n-1)}}{1}$

$9765625 = 5^{(n-1)}$

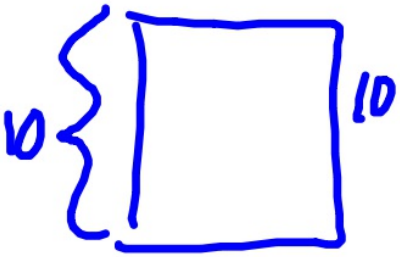
$\frac{\log 9765625}{\log 5} = \frac{(n-1)\log 5}{\log 5}$

$10 = n-1$

$11 = n$

Example 5:

$$a_n = a_1 \bullet r^{(n-1)}$$



Suppose you want a reduced copy of a photograph. The actual length of the photograph is 10 in. The smallest size the copier can make is 64% of the original. Find the length of the photograph after five reductions.

$a_1 = 10$
$r = .64$
$n = 5$

$a_5 = 10 \cdot (.64)^{(5-1)}$

$a_5 = 1.67 \text{ in}$

Geometric Mean

- Used to find the missing term of a geometric sequence
- The positive square root of the product of the two numbers

$$\text{geometric mean} = \sqrt{\text{product of the two numbers}}$$

Geometric Mean

Ex 10: Find the missing term of each geometric sequence

20, 40, 80, …

3, 7.5, 18.75, …

28, 378, 5103, …

$\sqrt{20 \cdot 80} = 40$

$\sqrt{3 \cdot 18.75} = 7.5$

$\sqrt{28 \cdot 5103} = 378$

Solutions

- 40
- 7.5
- 378

Determine if the sequence is geometric. If it is, find the common ratio.

1) $-1,\ 6,\ -36,\ 216, \ldots$

2) $-1,\ 1,\ 4,\ 8, \ldots$

5) $-2,\ -4,\ -8,\ -16, \ldots$

6) $1,\ -5,\ 25,\ -125, \ldots$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7) $a_n = 3^{n-1}$

8) $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8,\ r = -5$

16) $a_1 = 1,\ r = 2$