

Warm Up

Find the common ratio or difference for the following sequences. Write an explicit formula for the following sequences.

5, 7, 9, 11, 13, 15. . .

1, 3, 9, 27, 81, 243. . .

Answers

Write an explicit formula for the following sequences.

5, 7, 9, 11, 13, 15. . .

$$a_n = 2n + 3$$

1, 3, 9, 27, 81, 243. . .

$$a_n = 1 \cdot 3^{(n-1)}$$

8.3

Series and Summation Notation

Series

A series is the **sum** of the terms in a sequence.

$$3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 = 135$$

Series Formulas

Arithmetic
Series (*Sum*)

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric
Series (*Sum*)

$$S_n = \frac{a_1 \cdot (1 - r^n)}{(1 - r)}$$

Steps for Evaluating a Series

- 1) Determine if the terms in the series are arithmetic or geometric.
 - Common difference or common ratio?
- 2) Substitute the information you are given into the appropriate formula.
- 3) Calculator

Example 1

Evaluate a series with the terms 1, 7, 13, 19, 25
for the first 13 terms.

Example 2

Find the sum of the first 10 terms of the series with $a_1 = 6$ and $r = 2$.

You try!

Evaluate a series with the terms 2, 10, 18, 26, 34 for the first 15 terms.

Find the sum of the first 6 terms of the series with $a_1 = 7$ and $r = 1/2$.

Example 3

A philanthropist donates \$50 to the SPCA. Each year, he pledges to donate 12 dollars more than the previous year. In 8 years, what is the total amount he will have donated? (Hint: What is the price in 8 years?)

Summation Notation

Mathematicians are lazy! Instead of saying:

“Find the sum of the series denoted by $a_n = 3n + 2$ from the 3rd term to 7th term,”

they made up a symbol to deal with it.

Sigma!



I'm just a fancy way of saying, "Add everything up!"

Summation Notation

last
term

$$\sum_{n=3}^7 3n + 2$$

first
term

.

Example 4: Evaluating Using Summation Notation

$$\sum_{n=7}^{10} n - 3 =$$

$$\sum_{j=3}^5 \frac{1}{j} =$$

$$\sum_{k=1}^5 k^2 =$$

Writing Series in Summation Notation

Step 1) Determine the explicit formula.

Step 2) Identify the lower and upper limits.

Step 3) Write the series in summation notation.

Example 5

Use summation notation to write the series for the specified number of terms.

$$1 + 2 + 3 + \dots; t = 6$$

Example 6

Use summation notation to write the series for the specified number of terms.

$$3 + 8 + 13 + 18 + \dots; n = 9$$

Example 7

Use summation notation to write the series for the specified number of terms.

$$3 + 6 + 9 + \dots; n = 33$$

Evaluate each series.

A

13) $a_1 = 2, a_n = 122, n = 13$

14) $a_1 = -18, a_n = -102, n = 13$

15) $20 + 27 + 34 + 41\dots, n = 16$

16) $20 + 30 + 40 + 50\dots, n = 15$

17) $1 - 5 + 25 - 125\dots, n = 7$

18) $-3 - 6 - 12 - 24\dots, n = 9$

Find the number of terms in each sequence.

19) $a_1 = 19, a_n = 96, S_n = 690$

20) $a_1 = 16, a_n = 163, S_n = 4475$

21) $a_1 = -2, r = 5, S_n = -62$

22) $a_1 = 3, r = -3, S_n = -60$

rite each series as a sum. (In calculator, [Alpha] [Window] [2])

$$\sum_{m=1}^5 (4m^2 + 4)$$

$$2) \sum_{k=1}^5 (30 - k^2)$$

$$\sum_{n=1}^5 n$$

$$4) \sum_{m=1}^6 (50 - m)$$

$$\sum_{a=1}^6 (3a^2 - 2)$$

$$6) \sum_{m=1}^5 (100 - m)$$

Evaluate each series.

A

13) $a_1 = 2, a_n = 122, n = 13$

14) $a_1 = -18, a_n = -102, n = 13$

15) $20 + 27 + 34 + 41\dots, n = 16$

16) $20 + 30 + 40 + 50\dots, n = 15$

17) $1 - 5 + 25 - 125\dots, n = 7$

18) $-3 - 6 - 12 - 24\dots, n = 9$

Find the number of terms in each sequence.

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Rewrite each series as a sum. (In calculator, [Alpha] [Window] [2])

$$1) \sum_{m=1}^5 (4m^2 + 4)$$

$$2) \sum_{k=1}^5 (30 - k^2)$$

$$3) \sum_{n=1}^5 n$$

$$4) \sum_{m=1}^6 (50 - m)$$

$$5) \sum_{a=1}^6 (3a^2 - 2)$$

$$6) \sum_{m=1}^5 (100 - m)$$

