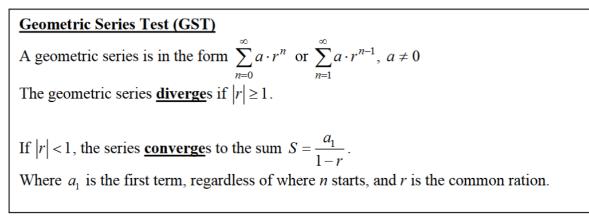
Tests for Convergence and Divergence

Geometric Series, nth Term Test for Divergence, and Telescoping Series



Example 8:

Using the GST, determine whether the following series converge or diverge. If the converge, find the sum.

(a) $\sum_{n=1}^{\infty} \frac{3}{2^n}$	(b) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$	(c) $\sum_{n=2}^{\infty} 3 \left(-\frac{1}{2}\right)^n$
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<u>nth Term Test for Divergence (ONLY)</u>	
If $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.	
(think about it, it should make perfect sense!)	

Example 9:

Determine whether the following series converge or diverge. If they converge, find their sum.

(a)
$$\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$$
 (b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$ (c) $\sum_{n=1}^{\infty} \frac{3^n-2}{3^n}$ (d) $\sum_{n=2}^{\infty} \frac{1}{(1.1)^n}$

A series such as $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots$ is called a **telescoping series** because it collapses to one term or just a few terms. If a series collapses to a finite sum, then it converges by the <u>**Telescoping Series**</u> <u>**Test**</u>.

Example 10:

Determine whether the following series converges or diverges. If they converge, find their sum.

(a)
$$\sum_{n=1}^{\infty} \left(\frac{1}{2n+1} - \frac{1}{2n+3} \right)$$
 (b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n+3}$

Integral Test and p-Series

Integral Test If *f* is Decreasing, Continuous, and Positive (Dogs Cuss in Prison!) for $x \ge 1$ AND $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x) dx$ either BOTH converge or diverge. Note 1: This does NOT mean that the series converges to the value of the definite integral!!!!!!! Note 2: The function need only be decreasing for all x > k for some $k \ge 1$. If the series converges to *S*, then the remainder, $R_n = |S - S_n|$ is bounded by $0 \le R_n \le \int_n^{\infty} f(x) dx$. (Not on AP exam, but on my exam.). This means $S \in [S_n, S_n + R_n]$.

Example 11:

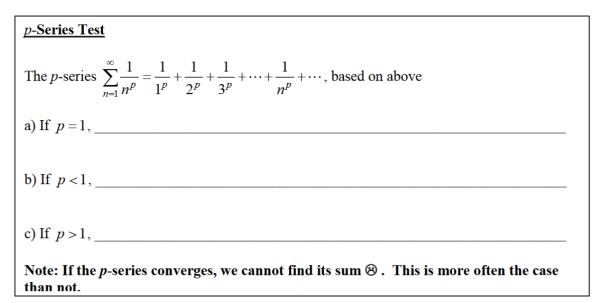
Determine whether the following series converge or diverge. If they converge, find an interval in which the sum resides using S_4 .

(a)
$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$
 (b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

<u>p-series</u>

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ is called a *p*-series, where *p* is a positive constant. For p = 1, the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$ is called the <u>harmonic series</u>.

Based on your experience with *p*-series and their reliance on the number one, fill in chart below.

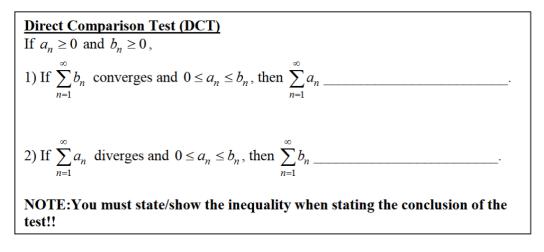


Example 13:

Determine of the following converges or diverges:

(a)
$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$$
 (b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n}}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$ (d) $\sum_{n=1}^{\infty} \frac{9999999999}{n^{1.000000001}}$

Based on your experience with improper integrals, again, fill in the chart below. **Comparison of Series**



Example 14:

Determine whether the following converge or diverge.

(a)
$$\sum_{n=1}^{\infty} \frac{n^3}{n^3 + 1}$$
 (b) $\sum_{n=1}^{\infty} \frac{1}{n^3 + 1}$ (c) $\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$

(d)
$$\sum_{n=4}^{\infty} \frac{1}{\sqrt{n-1}}$$
 (e) $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$ (f) $\sum_{n=2}^{\infty} \frac{1}{n^4 - 10}$