9.4 Power Series II: Geometric Series

Example 1:
First we'll do a quick review of geometric series. Geometric series are formed by multiplying by a common ratio r.

(a) Suppose I told you to start with $a_1 = 1$ and to let $r = \frac{2}{3}$, what geometric series would you write? What would the sum be?

(b) What if $a_1 = 1$ and $r = -\frac{2}{3}$?

(c) What if $a_1 = 1$ and $r = x$?

Example 2:

Verify your answer from Example 1(c) by finding the power series for $\frac{1}{1-x}$ centered at $c = 0$ by

(a) using Taylor's Rule
(b) using LOOOONG DIVISION.
(c) Find the radius and interval of convergence. Verify by graphing.

Example 3:

Find a power series for $\frac{1}{1+x}$ centered at $c = 0$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 4:

Find a power series that represents $\frac{x}{1+x}$ centered at $c = 0$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 5:

Find a power series for $f(x)=\dfrac{1}{1-x^2}$ centered at $c=0$, then find the interval of convergence. Find the first four nonzero terms and the general term.

When you replace x with a multiple of x, beware a change in the radius and interval of convergence. . .

Example 6:

Find a power series that represents $\frac{1}{1-2x}$ centered at $c = 0$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 7:

Find a power series for $g(x)=\dfrac{1}{4+x}$ centered at $c=0$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Sometimes we cannot center our function at $x = 0$. In this case, we must try to rewrite our function with the new center showing.

Example 8:

Find a power series that represents $\frac{1}{x}$ centered at $c = 1$, then find the interval of convergence. Include the first four nonzero terms and the general term.

Example 9:

(A bit of a booger) Find a power series for $h(x)=\frac{15}{2x-1}$, centered at $c=1$, then find the interval of convergence. Include the first four nonzero terms and the general term.