AP Calculus AB
Final Review Worksheet

I. **Find the following derivatives.**

1) $\frac{d}{dx}\sin x$

2) $\frac{d}{dx}\cos x$

3) $\frac{d}{dx}\tan x$

4) $\frac{d}{dx}\csc x$

5) $\frac{d}{dx}\sec x$

6) $\frac{d}{dx}\cot x$

7) $\frac{d}{dx}\left(e^x\right)$

8) $\frac{d}{dx}\left(2^x\right)$

9) $\frac{d}{dx}\ln x$

10) $\frac{d}{dx}\log_2 x$

11) $\frac{d}{dx}\arcsin x$

12) $\frac{d}{dx}\arccos x$

13) $\frac{d}{dx}\arctan x$

Find the following antiderivatives.

14) $\int x^{-1}\,dx$

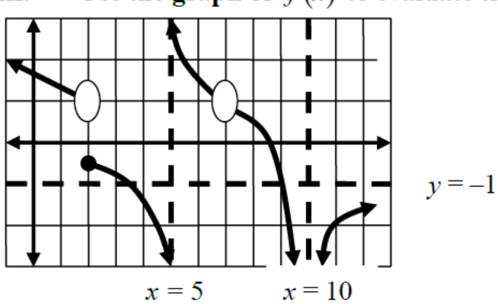
II. **Complete the following Trig Identities.**

1) $\sec^2 x =$

2) $\sin(2x) =$

3) $\cos(2x) =$

III. **Use the graph of $f(x)$ to evaluate the following limits**



(1) $\lim_{x\to 7} f(x) =$

(2) $\lim_{x\to 2^-} f(x) =$

(3) $\lim_{x\to 10} f(x) =$

(4) $\lim_{x\to 5} f(x) =$

(5) $\lim_{x\to 2} f(x) =$

(6) $\lim_{x\to \infty} f(x) =$

Evaluate the following limits.

(7) $\lim_{x\to 3}\frac{x-3}{x^2-9} =$

(8) $\lim_{x\to 0}\frac{(\sin 2x)}{2x} =$

(9) $\lim_{x\to 4}\frac{-2}{x^2-8x+16} =$

(10) $\lim_{x\to \infty}\frac{2x^2+3x}{4+3x^2} =$

(11) (CALC) $\lim_{x\to 0+}\left(1+\sqrt{x}\right)^{\frac{1}{\sqrt{x}}} =$

IV. (1) **What are the two versions of the definition of derivative at a point?**

Evaluate.

(2) $\lim_{h\to 0}\frac{\sin(\pi+h)-\sin\pi}{h} =$

(3) $\lim_{x\to e}\frac{3\ln x-3}{x-e} =$

V. (1) Determine if *f* is **continuous** at $x=2$. Justify your answer to using the **definition of continuity**.
(2) Determine if *f* is **differentiable** at $x=2$.

$$f(x)=\begin{cases} x^2, & x\le 2 \\ 2x, & x>2 \end{cases}$$

VI. Evaluate the derivatives of the following functions.

(1) $f(x)=x^3e^{2x}$ (2) $f(x)=\dfrac{\ln x}{x}$ (3) $f(x)=\sin^{-1}\left(\cos\left(x^2\right)\right)$

VII. Use implicit differentiation to find the following.

(1) Find $\dfrac{dy}{dx}$ if $4x^2y-3y=x^3$ (2) Find $\dfrac{d^2y}{dx^2}$ if $x^2+y^2=1$.

VIII. Second Derivative Test

Given $f'(a)=0$ (*f* has a stationary point). Then:

(a) If $f''(a)>0$, *f* is CU, and $x=a$ is a LMin.

(b) If $f''(a)<0$, *f* is CD, and $x=a$ is a LMax.

(c) If $f''(a)=0$, nothing can be determined.

(1) The slope on a curve at $(3,2)$ is 0. $\dfrac{d^2y}{dx^2}=\dfrac{(8y-3x)\left(3\dfrac{dy}{dx}-2\right)-(3y-2x)\left(8\dfrac{dy}{dx}-3\right)}{(8y-3x)^2}$. Does the curve have a local maximum, local minimum or neither at $(3,2)$?

IX. Derivatives of Inverses

If *f* and *g* are inverses, then $g'(x)=\dfrac{1}{f'(g(x))}$.

(1) Let *f* be the function defined by $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

X. Applications of Derivatives

(Note: some parts of the Free Response question below have integration concepts.)

(1) What is a normal line?
(2) For applications of graphs, remember your 'double" and "triple" charts.
(3) For applications of implicit differentiation, remember product rule.
(4) For Position/Velocity/Acceleration, remember PVA and the derivative – antiderivative relationships.
(5) For Slope Fields, plug in values into the given derivative and draw slopes.
(6) For Optimization (max/min), see Free Response: ** Always check both endpoints and all (stationary points) critical points when the problem asks for "absolute" max or min.
(7) A 13-foot ladder is leaning against a wall. The foot of the ladder is being pulled away from the wall at 4 feet per second. When the base is 5 feet from the wall, at what rate is the ladder going down the wall?

XI. **Theorems**

(1)Verify that the hypotheses of the Mean Value Theorem are satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the Mean Value Theorem for $f(x)=\sqrt{x+1}$ on $[0,3]$. (Algebra Slope – Calculus Slope)

(2)Verify that the hypotheses of Rolle's Theorem are satisfied on the given interval, and find all value of c in that interval that satisfy the conclusion of Rolle's Theorem for $f(x)=\cos x$ on $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$.

(3) Verify that the hypotheses of the Intermediate Value Theorem are satisfied on the given interval, and use the Intermediate Value Theorem to show that $f(x)=x^5+x^3-5x+2$ has a root in $[0,1]$.

(4)Verify that the hypotheses of the Extreme Value Theorem are satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the Extreme Value Theorem for $f(x)=x^2-3x$ on $[0,4]$.

XII. **Fundamental Theorem of Calculus**

(1) Evaluate $\frac{d}{dx}\left(\int_0^x \cos(t)\,dt\right)=$

(2) Evaluate $\frac{d}{dx}\left(\int_x^0 \cos(t)\,dt\right)=$

(3) Evaluate $\frac{d}{dx}\left(\int_0^{x^3} \cos(t)\,dt\right)=$

(4) DON'T FORGET TO ADD INITIAL CONDITION ON FREE RESPONSE GRAPH PROBLEMS!

XIII. **Area and Volume**

(1) Remember to make sure you are working in the correct variable for "top – bottom" and "right – left".

(2) Volume of Know Cross Sections Formulas:

(a) Equilateral Triangles $A=\frac{\sqrt{3}}{4}(\text{side})^2$ (b) Isosceles Triangles: $A=\frac{1}{4}(\text{hypotenuse})^2$

(c) Semicircles $A=\frac{1}{8}\pi(\text{diameter})^2$

XIV. **Riemann Sums**

(1) Find (a) the left sum, (b) the right sum, and (c) the trapezoidal sum for the given table. Use four subintervals.

0	4	6	7	10
6	9	2	5	3

(2) Find the midpoint sum using two subintervals.

0	5	10	15	20
7	3	8	2	9

XV. **Evaluate the following integrals.**

(1) $\int \frac{2x}{x^2+5}\,dx$ (2) $\int x^2\left(2+3x^3\right)^{-\frac{2}{5}}\,dx$

XVI. **Separable Differential Equations**

(1) The directions will likely say "Find the **PARTICULAR SOLUTION**".

(2) Solve $\frac{dy}{dx}=\frac{\sqrt{y^2+1}}{y}\cos x$ if $y=\sqrt{3}$ when $x=\pi$.

XVII. **Average Value**

(1) INTEGRAL OVER INTERVAL

(2) Find the average value of x^3 on $[1,5]$.

XVIII. **Miscellaneous**

(1) Remember ZOOMFIT on your calculator when the domain is given or can be obtained.

(2) See Free Response 2003: 3d. Problems like this MUST include a reference to time *b*.

(3) If *f* and *g* are inverse functions and $f(a)=b$. Then, $g'(x)=\frac{1}{f'(g(x))}$ where the right side is defined.

(4) Inflection points occur ONLY where concavity changes and the point is defined ON the FUNCTION.

(5) To graph a derivative of a function, put the function in Y_1. Then put $nDeriv(Y_1,X,X)$ in Y_2.

(6) To find $f'(a)$ on the calculator, $nDeriv(Y_1,X,a)$. (Or use the 2nd CALC menu.)

(7) To integrate from *a* to *b* on the calculator, $fnInt(Y_1,X,a,b)$

(8) See Free Response 2002: #2, 2004: #1

(9) REMEMBER: If a question asks WHERE a critical point, stationary point, or inflection point is, give ONLY the *x*-value. If a question asks for the VALUE of one of these points, give only the *y*-value.

(10) Remember: $\int v(t)\,dt$ is the net change in distance and $\int |v(t)|\,dt$ is the total distance traveled.

(11) Remember to include all units in descriptions.

(12) Remember that $f''(x)>0$ means that *f* is Concave Up **AND** that f' is increasing.

(13) Remember to state that the function is continuous if you are using one of "THE" theorems. Also, state that the function is differentiable if you are using MVT or Rolles.

(14) Remember: New Amount = Initial Condition + Integrate a Rate

$$f(b)=f(a)+\int_a^b f'(x)\,dx$$