Power Functions

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"Mathematics expresses values that reflect the cosmos, including orderliness, balance, harmony, logic, and abstract beauty."
~Deepak Chopra

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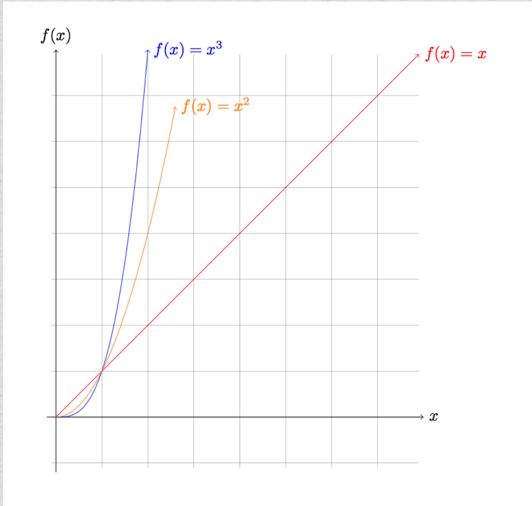
A power function is defined to be a function with a constant exponential (or power). The general form of a power function is $f(x) = ax^b$, where a and b are constants. Moreover, we have to restrict the domain of this function to be greater than zero, otherwise there may be non-real solutions. This means that x must be a positive real number. Great! Our life just got a lot easier because the graph will only appear in the first and fourth quadrant of the coordinate plane.

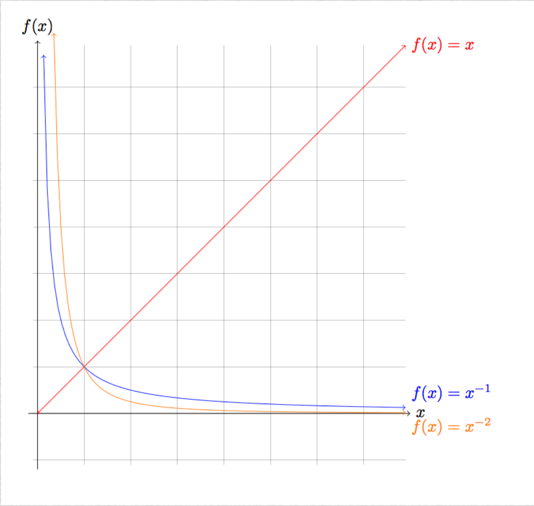
Properties of Power Functions

We can essential classify power functions in to two groups: the first groups consist of all the positive powers and naturally the second group is all the negative powers. To best visualize these two groups, we can look at the graphs of these functions. In the figure below, you can see the graphs of several positive powers of the function $f(x) = x^b$ namely, when $b = 1$, 2, and 3. Here we see that the higher the power, the faster it goes to infinity.



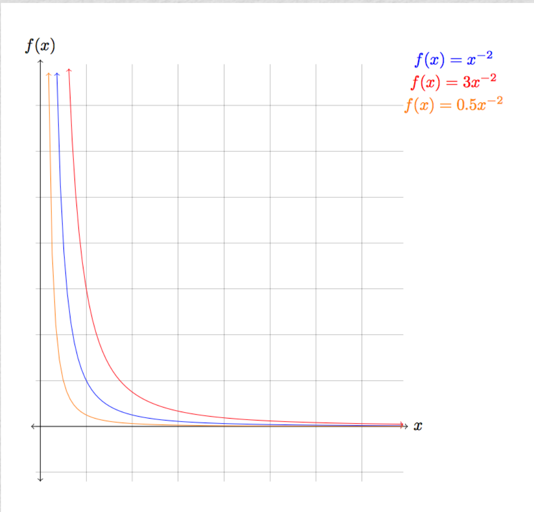
In this illustration, we see that $f(x) = x^3$ grows much faster than the other two functions. *Click this image for a full screen view.*

We now wish to illustrate how negative exponents affect the graphs of power functions. Below you will see a similar illustration, but this time with the exponents $b = -2, \ -1$, and 1. Here we can observe that the more negative the power, the faster the function goes to zero.



Here we see that more negative the power, the faster it goes to zero.

Naturally, the next question we can ask ourselves is, “What does the value of a do to the graph of the function?” Again, we rely on the graphs to help us answer this question. Just as in all other function, the coefficient can cause the function to increase or decrease more rapidly, or reflect it over the x-axis.



Picture here is what a coefficient does to a power function with a negative power.

In the case of negative powers, as we see in the figure, if the coefficient is greater than 1 (the red function), then it will cause the function to go to zero slower than if it is a fraction between zero and one (the orange function). In the case of a fraction between zero and one, the function will decrease quickly.

Recap

For a power function $f(x) = ax^b$, where x is positive:

1. If a and b are positive, then the function will increase to infinity rapidly depending on how large a and b are;
2. If a is positive and b is negative, then the function will decrease to zero;
3. If a is negative and b is positive, then the function will decrease to infinity rapidly depending on how negative a is;
4. If a and b are negative, then the function will increase to zero.

Exercises

1. Consider the power function $y = ax^3$. On the same screen in your calculator, make graphs of y versus x for $a = \{-1, 1, 2, 3\}$. Reflect on how the coefficient a affects the graph when the power is positive.

2. Consider the power function $y = ax^{-3}$. On the same screen in your calculator, make graphs of y versus x for $a = \{-1, 1, 2, 3\}$. Reflect on how the coefficient a affects the graph when the power is negative.

3. Consider the power function $y = x^{\frac{1}{k}}$. On the same screen in your calculator, make graphs of y versus x for $k = \{1, 2, 3, 4\}$. How does the value of k transform the graph?

4. Consider the power function $y = x^{\frac{k}{2}}$. On the same screen in your calculator, make graphs of y versus x for $k = \{1, 2, 3, 4\}$. How does the value of k transform the graph?

5. The equation $y = 3x^{\frac{1}{3}}$ is graphed on the coordinate plane. How would increasing the denominator of the power transform the graph of the function? [*Hint: Use the information you found from question 3.*]

6. The equation $y = x^{\frac{2}{3}}$ is graphed on the coordinate plane. How would increasing the numerator of the power transform the graph of the function? [*Hint: Use the information you found from question 4.*]

7. The mass-luminosity, L, of a main-sequence star is given by the relationship $L = M^{3.5}$, where M is the mass of the star. If a main-sequence star has a mass of approximately 1.6, what would be its mass-luminosity?

8. According to Newton's law of gravitation, the gravitational attraction between two objects is inversely proportional to the square of the distance from their centers. More generally, the force, F, between the objects is given by the power function $F = kd^{-2}$, where k is the correlation coefficient.

   a. Suppose the force of gravity causes two large planetary objects to begin moving towards one another. What is the effect on the gravitational force if the distance between them is halved?

   b. Suppose two large asteroids, whose centers are 500 kilometers apart, have a gravitational force of 1,400,000 newtons between them. What is the correlation coefficient?

   c. Using this correlation coefficient, in part b, find the gravitational force if the distance between their centers is increased to 700 kilometers.

   d. Using the information from parts b and c (or using the graph of the function with the value of k that you found, determine what happens to the gravitational force as the distance between the asteroids increases.