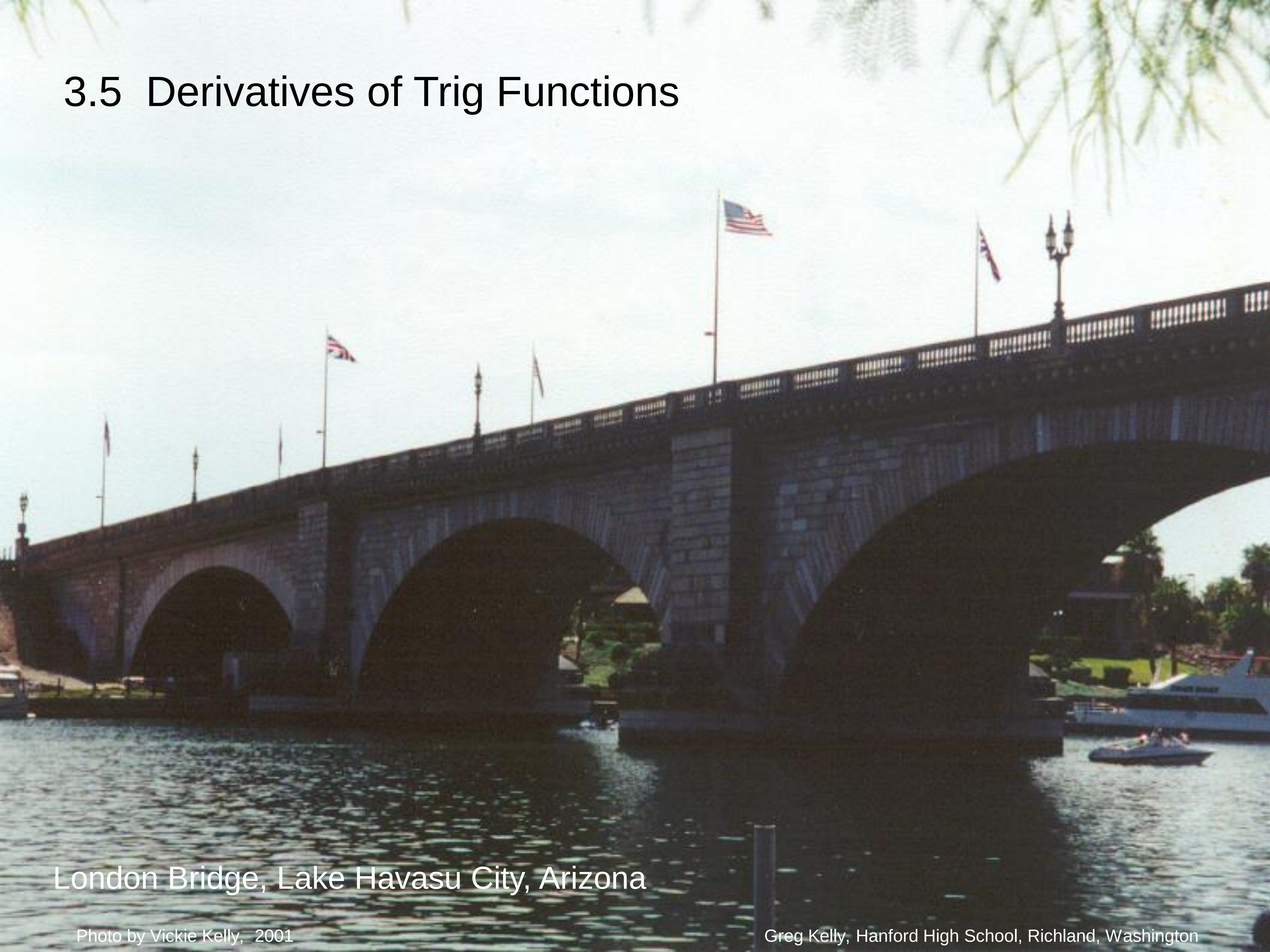
3.5 Derivatives of Trig Functions



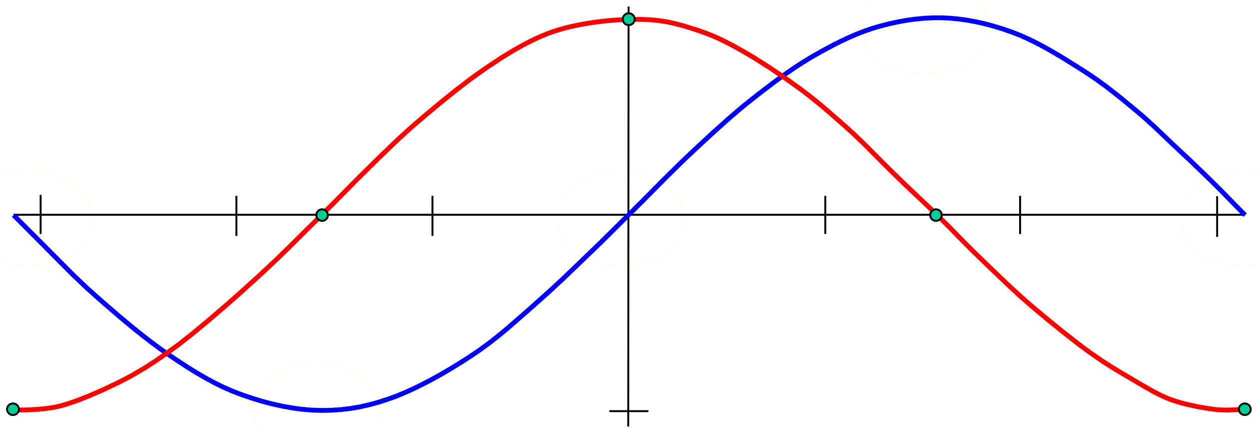
London Bridge, Lake Havasu City, Arizona

Photo by Vickie Kelly, 2001

Greg Kelly, Hanford High School, Richland, Washington

Consider the function $y = \sin(\theta)$

We could make a graph of the slope:



θ	slope
$-\pi$	-1
$-\frac{\pi}{2}$	0
0	1
$\frac{\pi}{2}$	0
π	-1

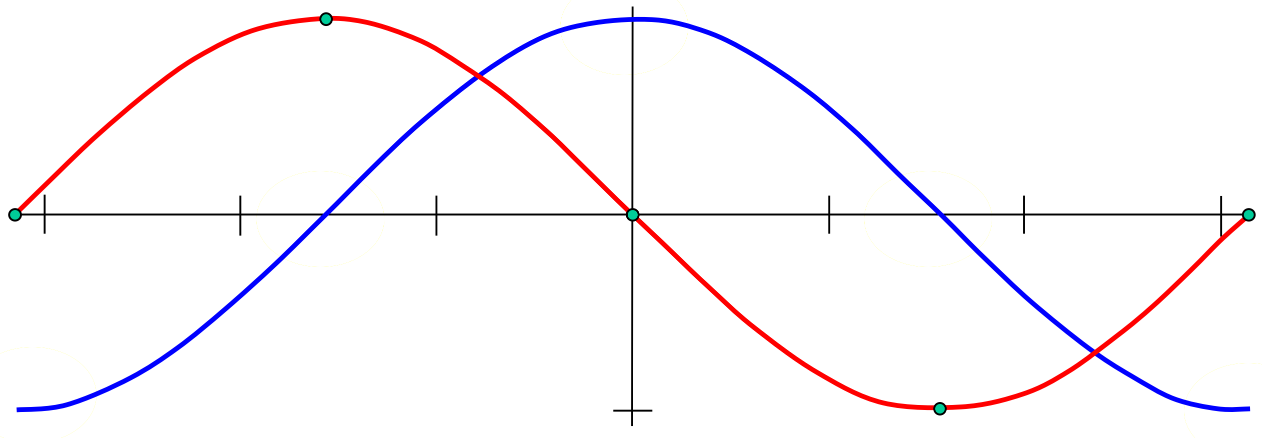
Now we connect the dots!

The resulting curve is a cosine curve.

$$\frac{d}{dx}\sin(x) = \cos x$$

→

We can do the same thing for $y = \cos(\theta)$



The resulting curve is a sine curve that has been reflected about the x-axis.

θ	slope
$-\pi$	0
$-\frac{\pi}{2}$	1
0	0
$\frac{\pi}{2}$	-1
π	0

$$\frac{d}{dx}\cos(x) = -\sin x$$

→

We can find the derivative of tangent *x* by using the quotient rule.

$$\frac{d}{dx}\tan x$$

$$\frac{d}{dx}\frac{\sin x}{\cos x}$$

$$\frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x}$$

$$\sec^2 x$$

$$\frac{d}{dx}\tan(x) = \sec^2 x$$

→

Derivatives of the remaining trig functions can be determined the same way.

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$$

π

Higher Order Derivatives:

$$y' = \frac{dy}{dx}$$ is the first derivative of y with respect to x.

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\frac{dy}{dx} = \frac{d^2 y}{dx^2}$$ is the second derivative.

(y double prime)

$$y''' = \frac{dy''}{dx}$$ is the third derivative.

$$y^{(4)} = \frac{d}{dx} y'''$$ is the fourth derivative.

We will learn later what these higher order derivatives are used for.

π

Assignment
p. 146 # 1-10

When finished go to http://archives.math.utk.edu/visual.calculus/2/trig.1/ and write down the six trig derivatives then complete the first five product rule problems and the first five quotient rule problems.