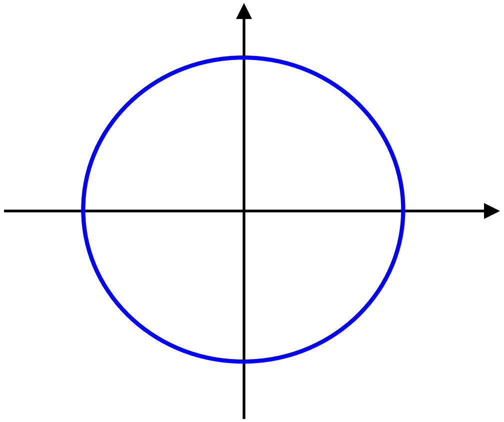
Warm-up - Trig Worksheet #7 and #8

7) $f(x) = \sin^3 x^5$

8) $f(x) = \cos\left(-3x^2 + 2\right)^2$


3.7 Implicit Differentiation
Niagara Falls, NY & Canada
Photo by Vickie Kelly, 2003
Greg Kelly, Hanford High School, Richland, Washington

$$x^2 + y^2 = 1$$



This is not a function, but it would still be nice to be able to find the slope.

$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}1$$

← Do the same thing to both sides.

Note use of chain rule.

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

→

$$2y = x^2 + \sin y$$ ← This can't be solved for y.

$$\frac{d}{dx}2y = \frac{d}{dx}x^2 + \frac{d}{dx}\sin y$$

$$2\frac{dy}{dx} = 2x + \cos y\frac{dy}{dx}$$

$$2\frac{dy}{dx} - \cos y\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}\left(2 - \cos y\right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

This technique is called <u>implicit differentiation.</u>

1. Differentiate both sides w.r.t. x.
2. Solve for $\dfrac{dy}{dx}$.

→

Find the equations of the lines tangent and normal to the curve $x^2 - xy + y^2 = 7$ at $(-1, 2)$.

We need the slope. Since we can't solve for y, we use implicit differentiation to solve for $\frac{dy}{dx}$.

$$x^2 - xy + y^2 = 7$$

$$2x - \left[x\frac{dy}{dx} + y \right] + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$(2y - x)\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$m = \frac{2 - 2(-1)}{2 \cdot 2 - (-1)} = \frac{2 + 2}{4 + 1} = \frac{4}{5}$$

→

Find the equations of the lines tangent and normal to the curve $x^2 - xy + y^2 = 7$ at $(-1, 2)$.

$$m = \frac{4}{5}$$

tangent:

$$y - 2 = \frac{4}{5}(x + 1)$$

$$y - 2 = \frac{4}{5}x + \frac{4}{5}$$

$$\boxed{y = \frac{4}{5}x + \frac{14}{5}}$$

normal:

$$y - 2 = -\frac{5}{4}(x + 1)$$

$$y - 2 = -\frac{5}{4}x - \frac{5}{4}$$

$$\boxed{y = -\frac{5}{4}x + \frac{3}{4}}$$

→

Higher Order Derivatives

Find $\dfrac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 7$.

$$2x^3 - 3y^2 = 7$$

$$6x^2 - 6y\,y' = 0$$

$$-6y\,y' = -6x^2$$

$$y' = \frac{-6x^2}{-6y}$$

$$y' = \frac{x^2}{y}$$

$$y'' = \frac{y \cdot 2x - x^2 y'}{y^2}$$

$$y'' = \frac{2x}{y} - \frac{x^2}{y^2}\,y'$$

$$y'' = \frac{2x}{y} - \frac{x^2}{y^2} \cdot \frac{x^2}{y}$$

Substitute y' back into the equation.

$$\boxed{y'' = \frac{2x}{y} - \frac{x^4}{y^3}}$$

→

p.162   #1-7 odd, 10, 11, 19, 24, 27, 30 p.162   #2-8 even, 9, 12, 20, 22, 29

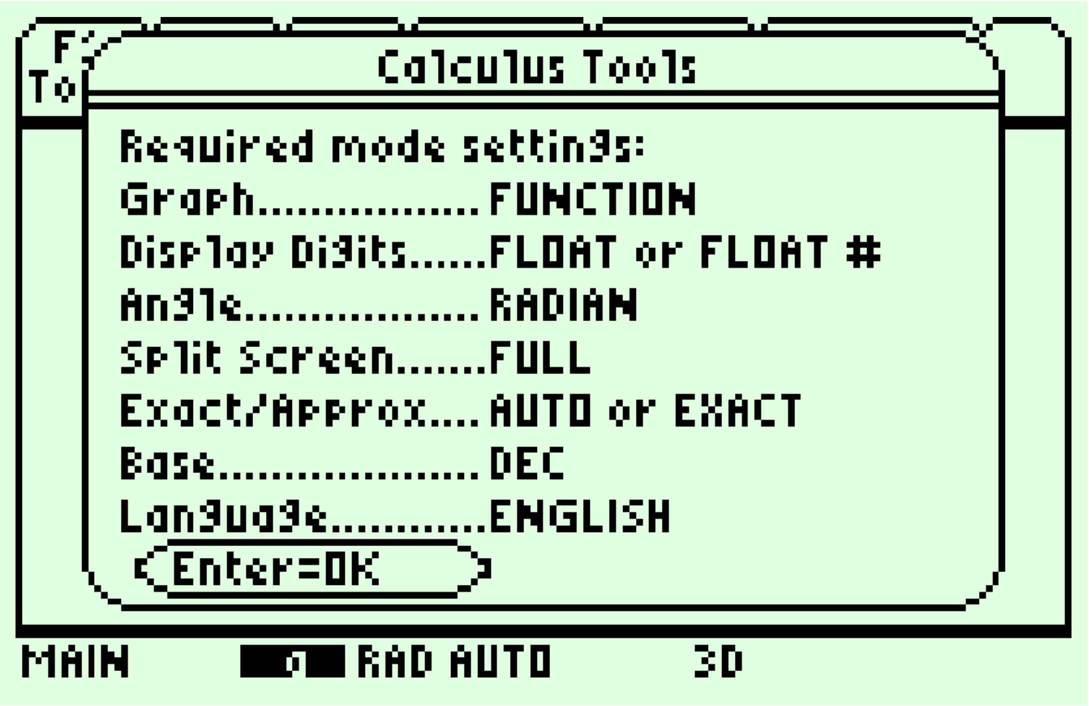
Now let's do one on the TI-89:

$$2x^3 - 3y^2 = 7 \quad \text{Find } \frac{d^2y}{dx^2}.$$

◆ APPS

Select Calculus Tools and press ENTER .

If you see this screen, press ENTER , change the mode settings as necessary, and press ◆ APPS again.



→

Now let's do one on the TI-89:

$$2x^3 - 3y^2 = 7 \quad \text{Find } \frac{d^2y}{dx^2}.$$

◆ APPS

Select Calculus Tools and press ENTER.

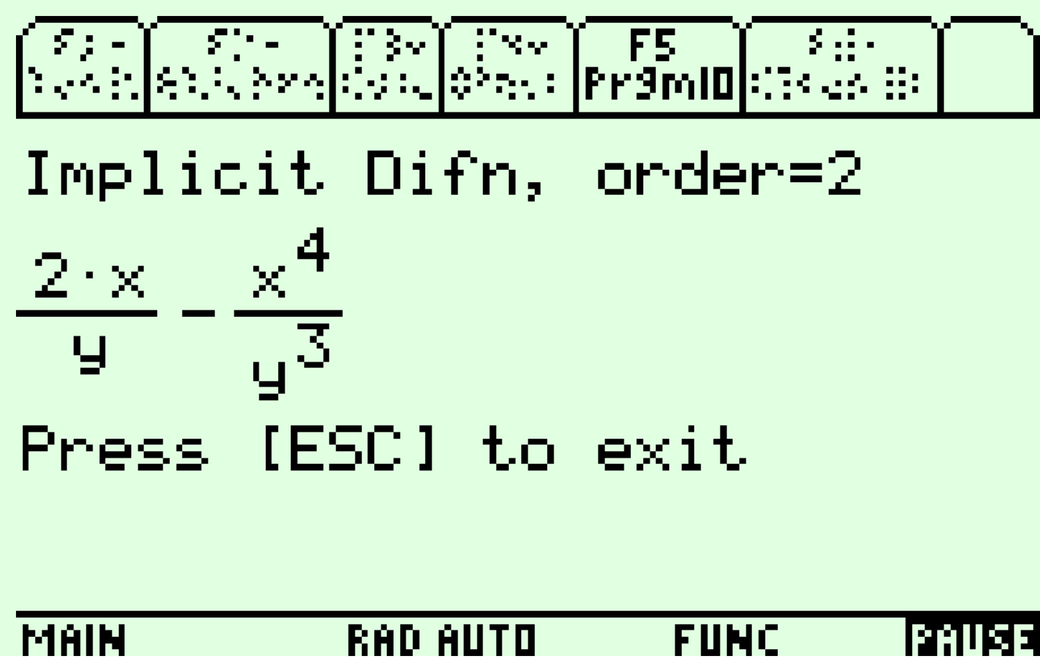
Press F2 (Deriv)

Press 4 (Implicit Difn)

Enter the equation.
(You may have to unlock the *alpha* mode.)

Set the order to 2.

Press ENTER.



Press **ESC** and then **HOME** to return your calculator to normal.