4.1 Extreme Values of Functions

Greg Kelly, Hanford High School
Richland, Washington

The textbook gives the following example at the start of chapter 4:

The mileage of a certain car can be approximated by:

$$m(v) = 0.00015v^3 - 0.032v^2 + 1.8v + 1.7$$

At what speed should you drive the car to obtain the best gas mileage?

Of course, this problem isn't entirely realistic, since it is unlikely that you would have an equation like this for your car.

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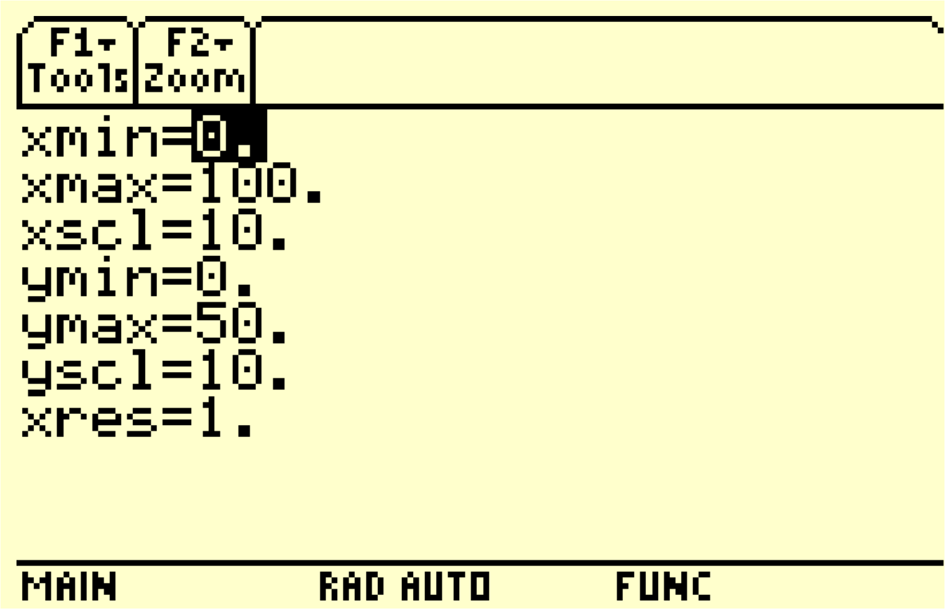
We could solve the problem graphically:

$$m(v) = 0.00015v^3 - 0.032v^2 + 1.8v + 1.7$$



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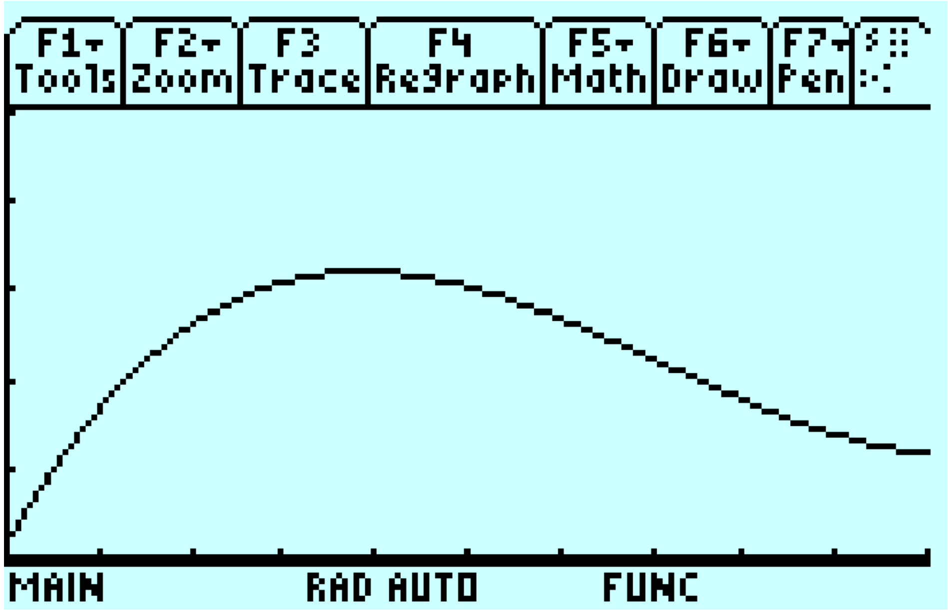

On the TI-89, we use F5 (math), 4: Maximum, choose lower and upper bounds, and the calculator finds our answer.

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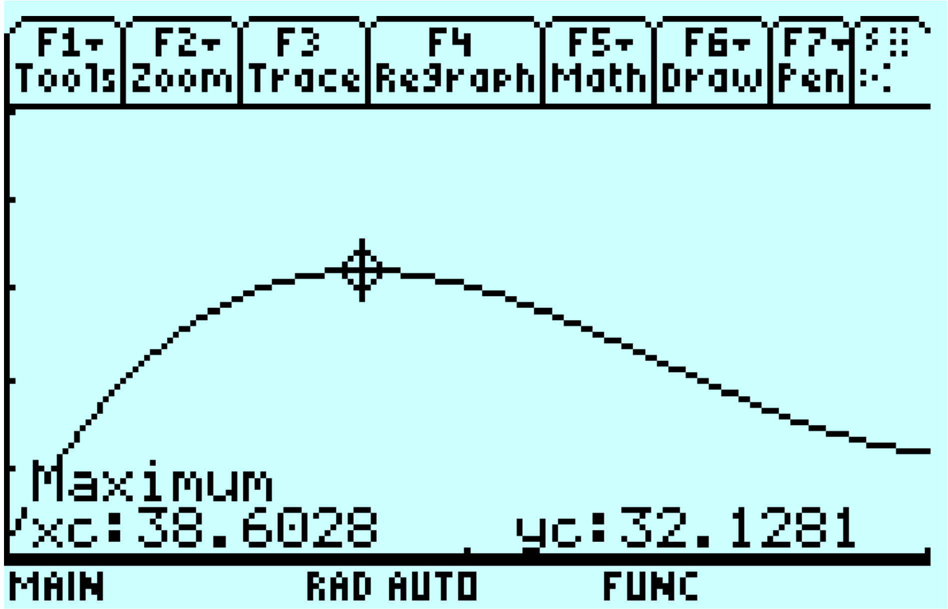

The car will get approximately 32 miles per gallon when driven at 38.6 miles per hour.

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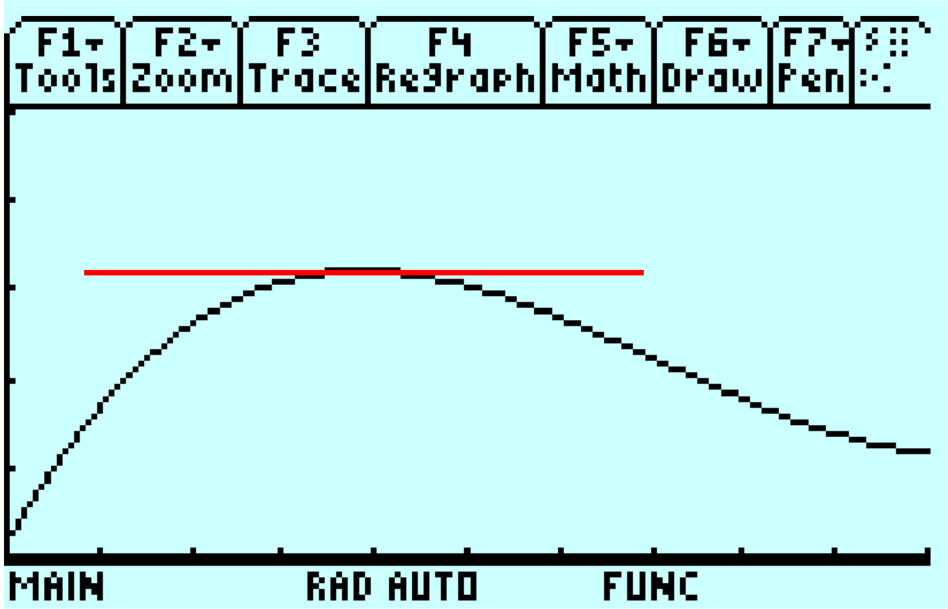
$$m(v) = 0.00015v^3 - 0.032v^2 + 1.8v + 1.7$$

Notice that at the top of the curve, the horizontal tangent has a slope of zero.



Traditionally, this fact has been used both as an aid to graphing by hand and as a method to find maximum (and minimum) values of functions.

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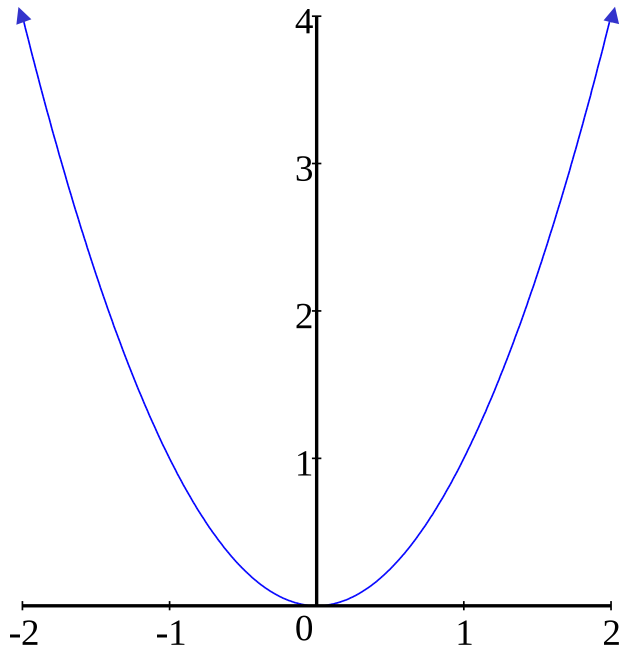
Even though the graphing calculator and the computer have eliminated the need to routinely use calculus to graph by hand and to find maximum and minimum values of functions, we still study the methods to increase our understanding of functions and the mathematics involved.

Absolute extreme values are either maximum or minimum points on a curve.

They are sometimes called global extremes.

They are also sometimes called absolute extrema. (*Extrema* is the plural of the Latin *extremum*.)

Extreme values can be in the interior or the end points of a function.



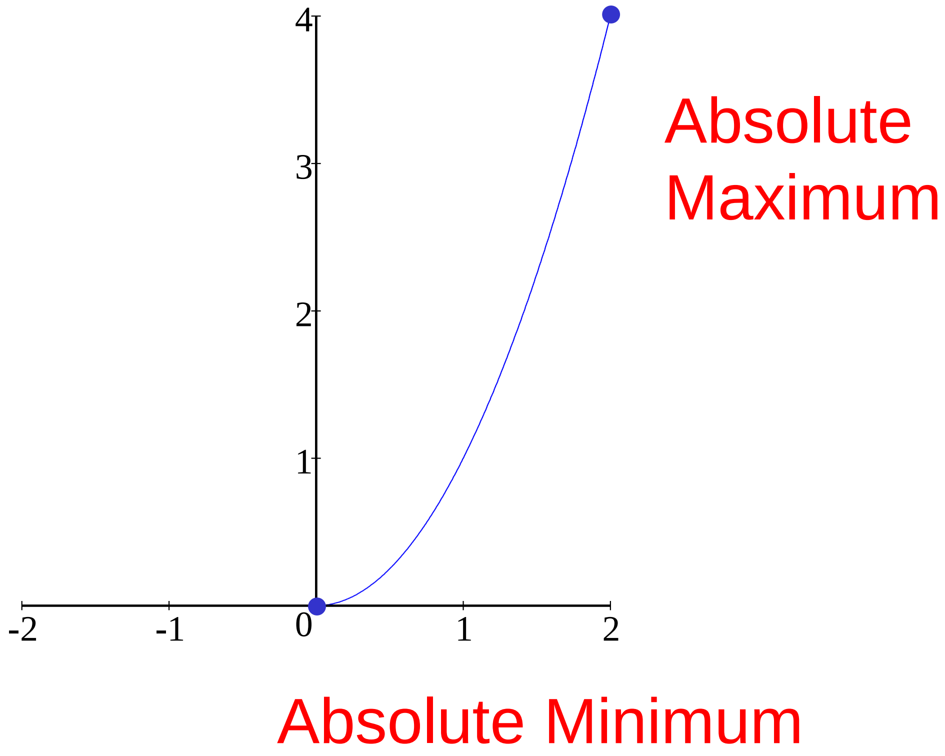
No Absolute Maximum

Absolute Minimum

$$y = x^2$$

$$D = (-\infty, \infty)$$

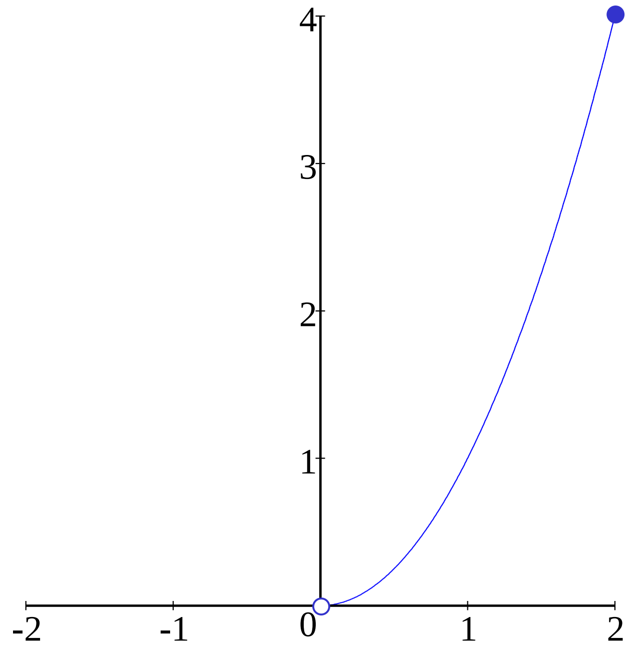
→



$$y = x^2$$

$$D = [0, 2]$$

→



Absolute Maximum

No Minimum

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→

$$y = x^2$$

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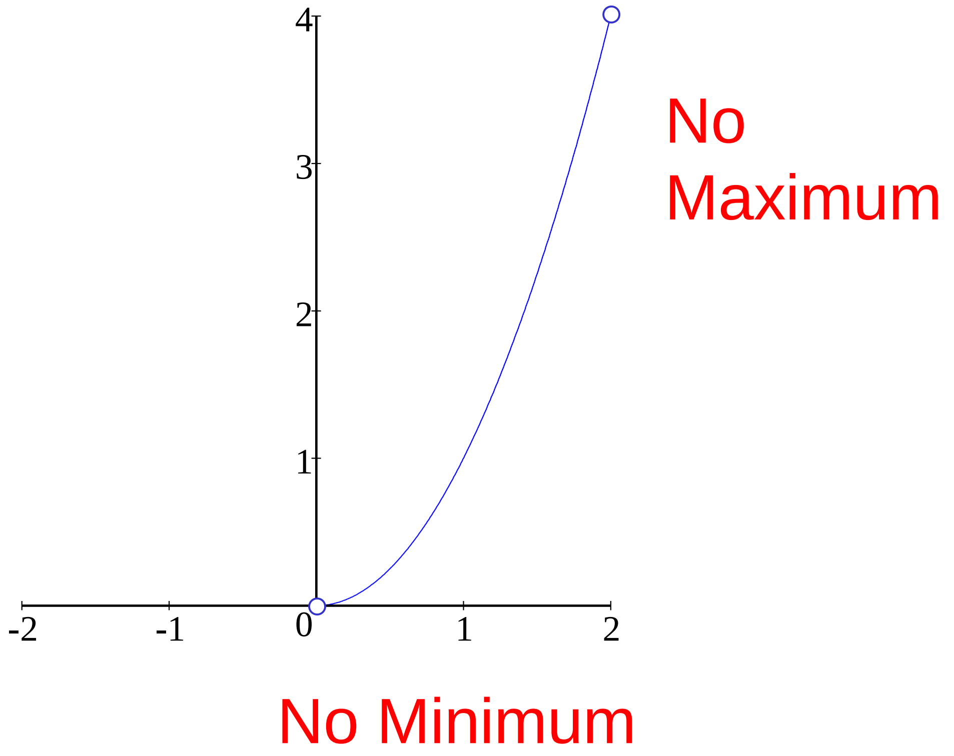

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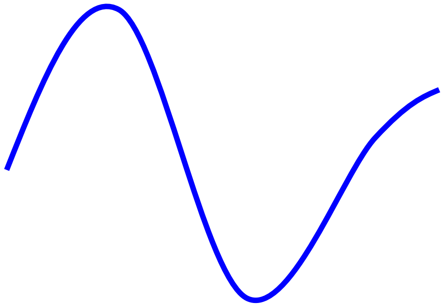
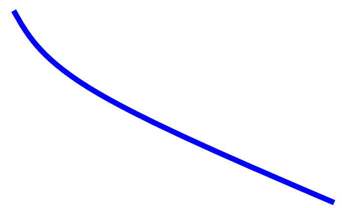
Extreme Value Theorem:

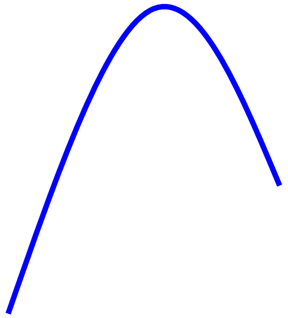
If f is continuous over a closed interval, then f has a maximum and minimum value over that interval.



Maximum & minimum at interior points



Maximum & minimum at endpoints



Maximum at interior point, minimum at endpoint

Local Extreme Values:

A local maximum is the maximum value within some open interval.

A local minimum is the minimum value within some open interval.

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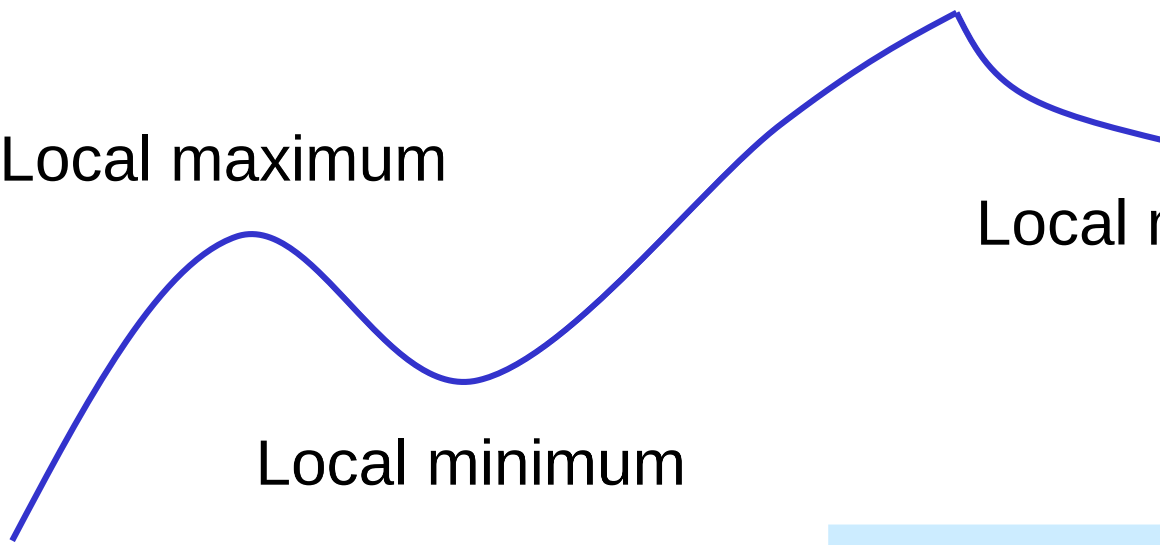
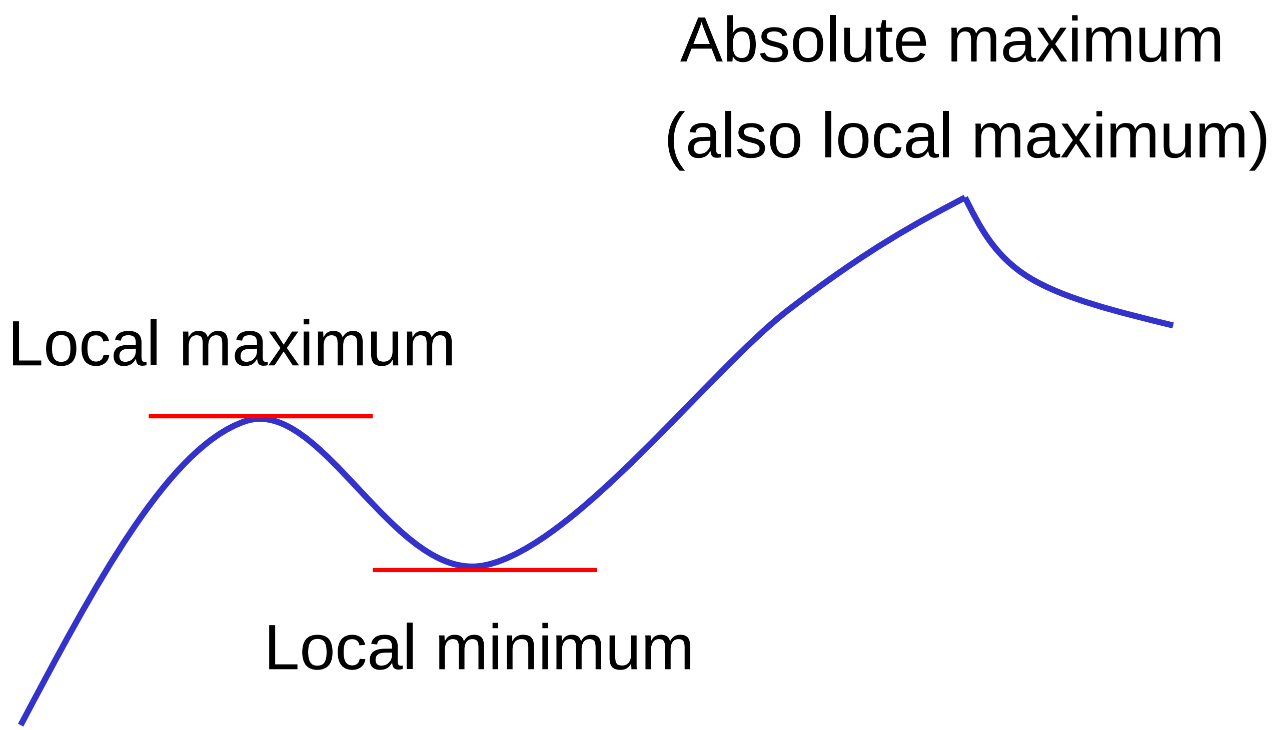


Local extremes are also called <u>relative extremes</u>.



Notice that local extremes in the interior of the function occur where f' is zero or f' is undefined.

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Local Extreme Values:

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c, then

$$f'(c) = 0$$

→

Critical Point:

A point in the domain of a function f at which $f' = 0$ or f' does not exist is a **critical point** of f.

Note:
Maximum and minimum points in the interior of a function always occur at critical points, but critical points are not always maximum or minimum values.

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EXAMPLE 3 FINDING ABSOLUTE EXTREMA

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

$$f(x) = x^{2/3}$$

There are no values of x that will make the first derivative equal to zero.

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$$

The first derivative is undefined at x=0, so (0,0) is a critical point.

$$f'(x) = \frac{2}{3\sqrt[3]{x}}$$

Because the function is defined over a closed interval, we also must check the endpoints.

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$$f(x) = x^{2/3} \qquad D = [-2, 3]$$

At: $x = 0 \quad f(0) = 0$

To determine if this critical point is actually a maximum or minimum, we try points on either side, without passing other critical points.

$$f(-1) = 1 \qquad f(1) = 1$$

Since 0<1, this must be at least a local minimum, and possibly a global minimum.

At: $x = -2 \quad f(-2) = (-2)^{\frac{2}{3}} \approx 1.5874$

At: $x = 3 \quad f(3) = (3)^{\frac{2}{3}} \approx 2.08008$

→

$$f(x) = x^{2/3} \qquad D = [-2, 3]$$

At: $x = 0 \quad f(0) = 0$

Absolute minimum: $(0, 0)$

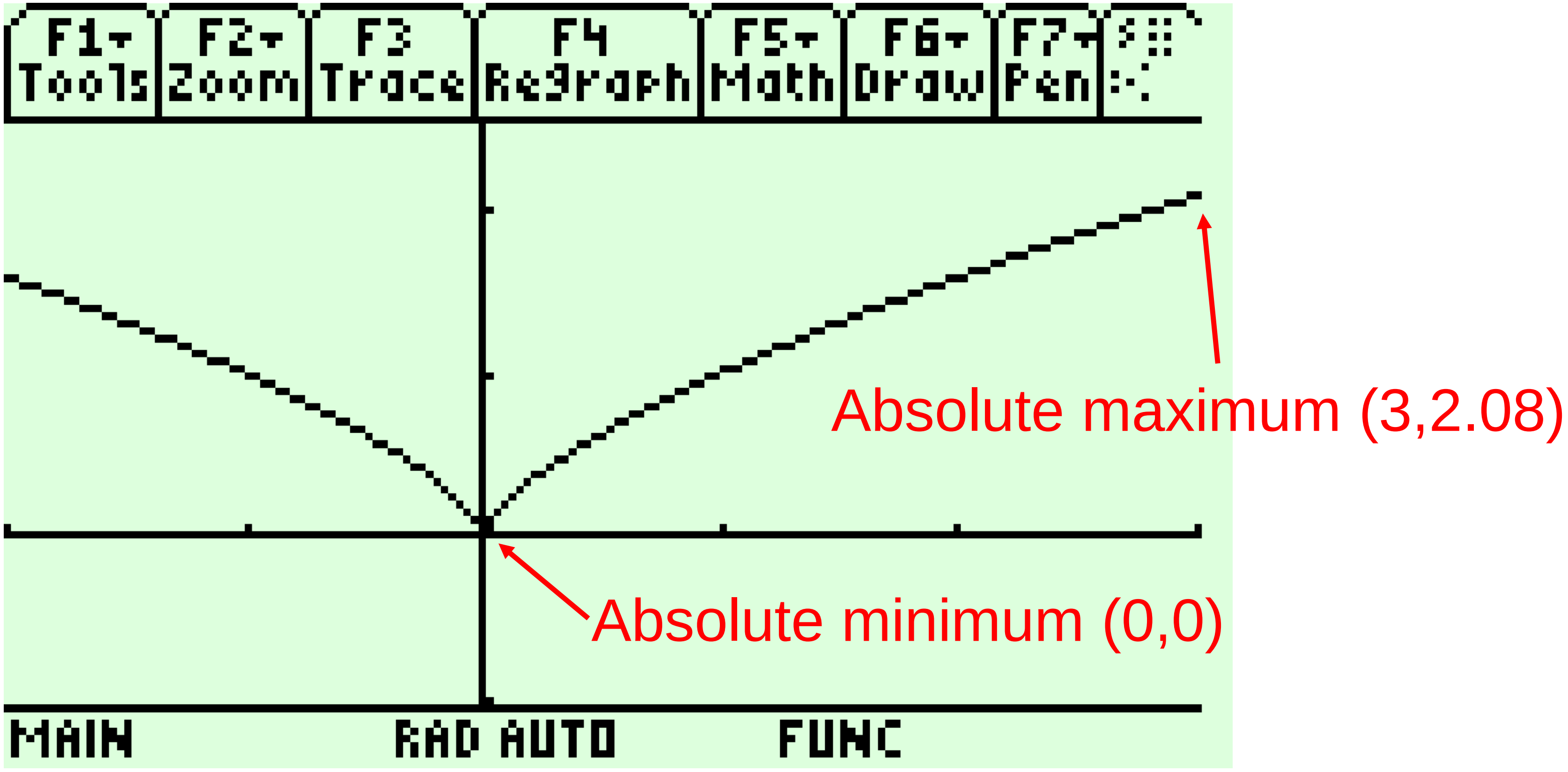
Absolute maximum: $(3, 2.08)$

$$f(-1) = 1 \qquad f(1) = 1$$

At: $x = -2 \quad f(-2) = (-2)^{\frac{2}{3}} \approx 1.5874$

At: $x = 3 \quad f(3) = (3)^{\frac{2}{3}} \approx 2.08008$

$\rightarrow$


F1▾
Tools
F2▾
Zoom
F3
Trace
F4
ReGraph
F5▾
Math
F6▾
Draw
F7▾
Pen
Absolute maximum (3,2.08)
Absolute minimum (0,0)
MAIN
RAD AUTO
FUNC

$$f(x) = x^{2/3}$$

→

Finding Maximums and Minimums Analytically:

1. Find the derivative of the function, and determine where the derivative is zero or undefined. These are the critical points.

2. Find the value of the function at each critical point.

3. Find values or slopes for points between the critical points to determine if the critical points are maximums or minimums.

4. For closed intervals, check the end points as well.

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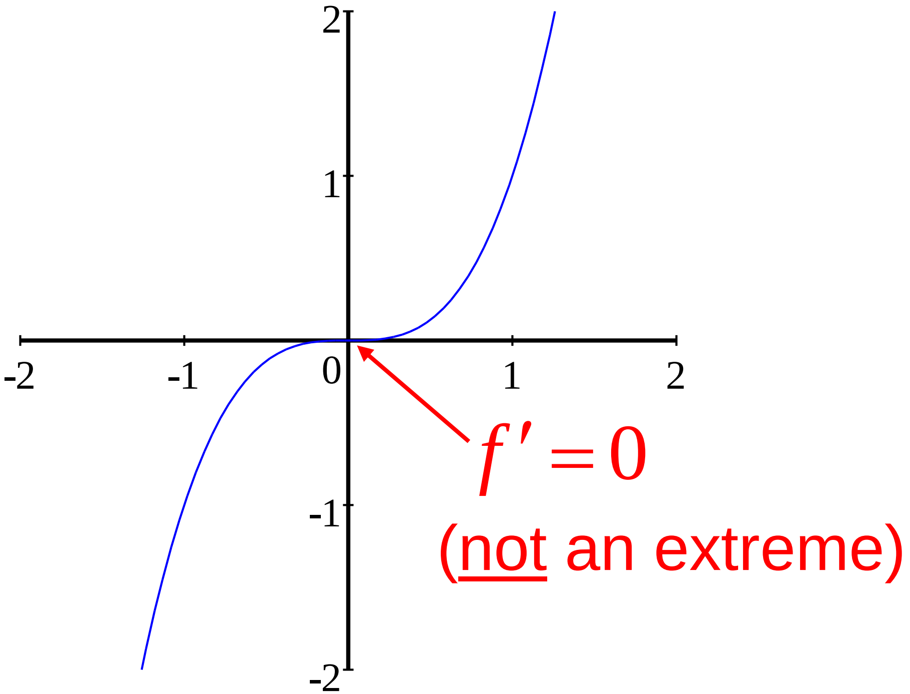
Critical points are not always extremes!

$$y = x^3$$



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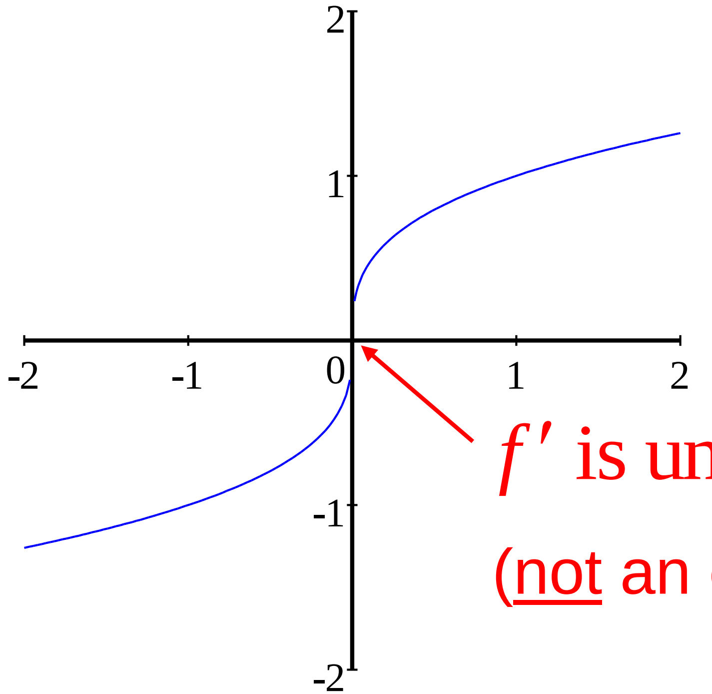
$y = x^{1/3}$



f' is undefined.

(<u>not</u> an extreme)

HW: p 193-194 Exercises #1-10