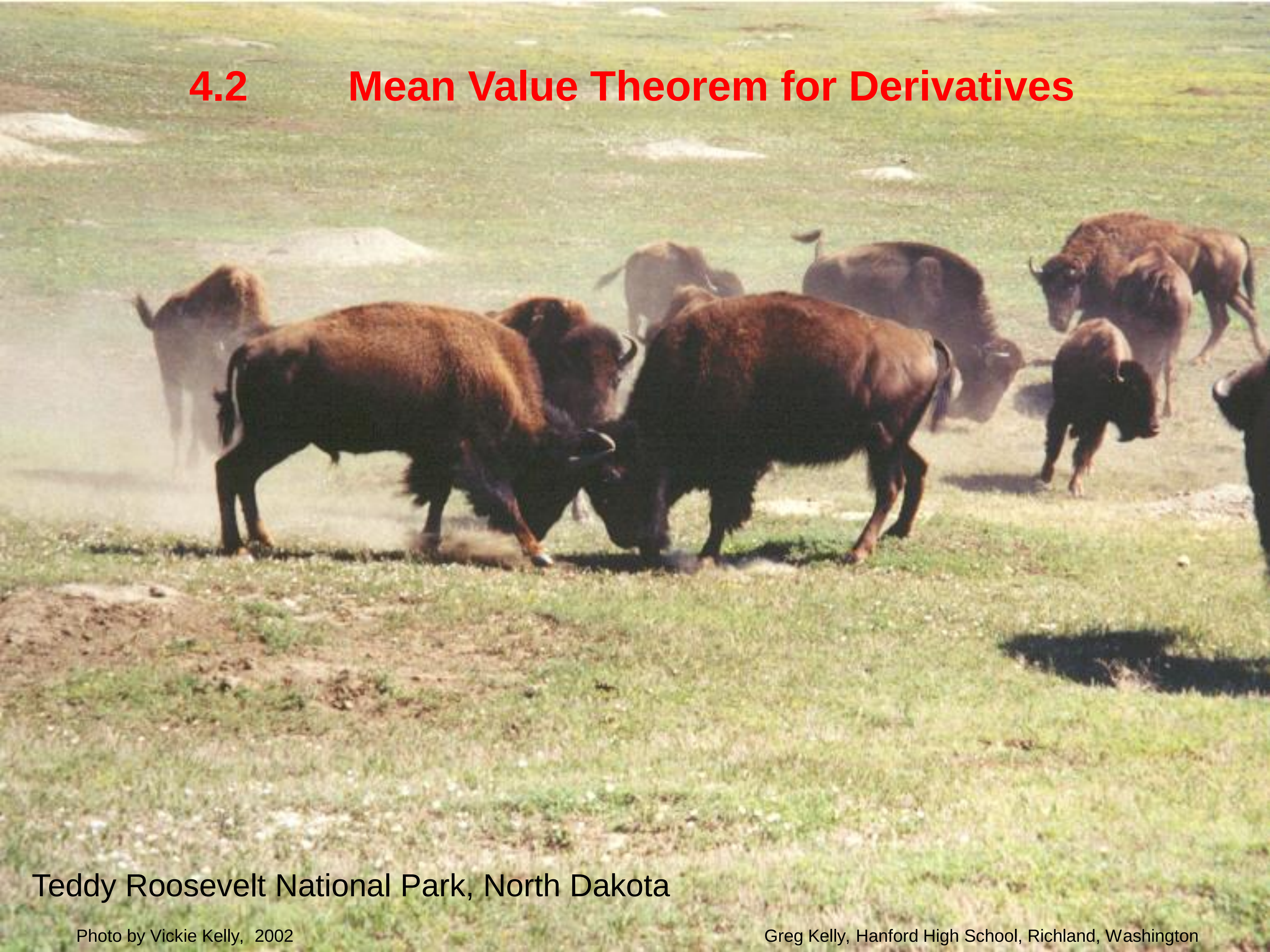
4.2 Mean Value Theorem for Derivatives



Teddy Roosevelt National Park, North Dakota

Photo by Vickie Kelly, 2002

Greg Kelly, Hanford High School, Richland, Washington

Mean Value Theorem for Derivatives

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The Mean Value Theorem only applies over a **closed interval.**

→

Mean Value Theorem for Derivatives

If $f(x)$ is a differentiable function over $[a,b]$, then at some point between a and b:

$$\frac{f(b)-f(a)}{b-a}=f'(c)$$

The Mean Value Theorem says that **at some point in the closed interval, the actual slope equals the average slope**.

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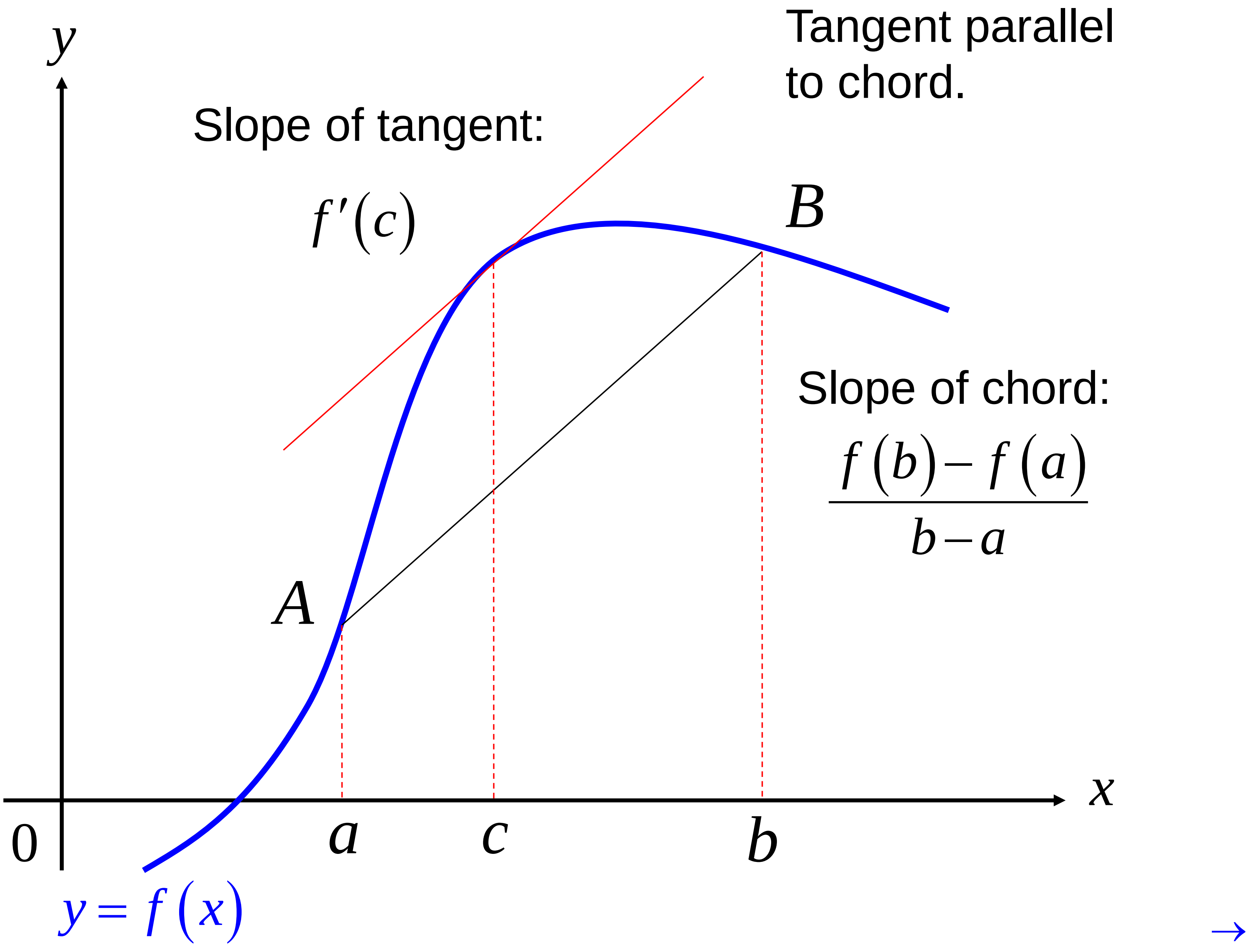
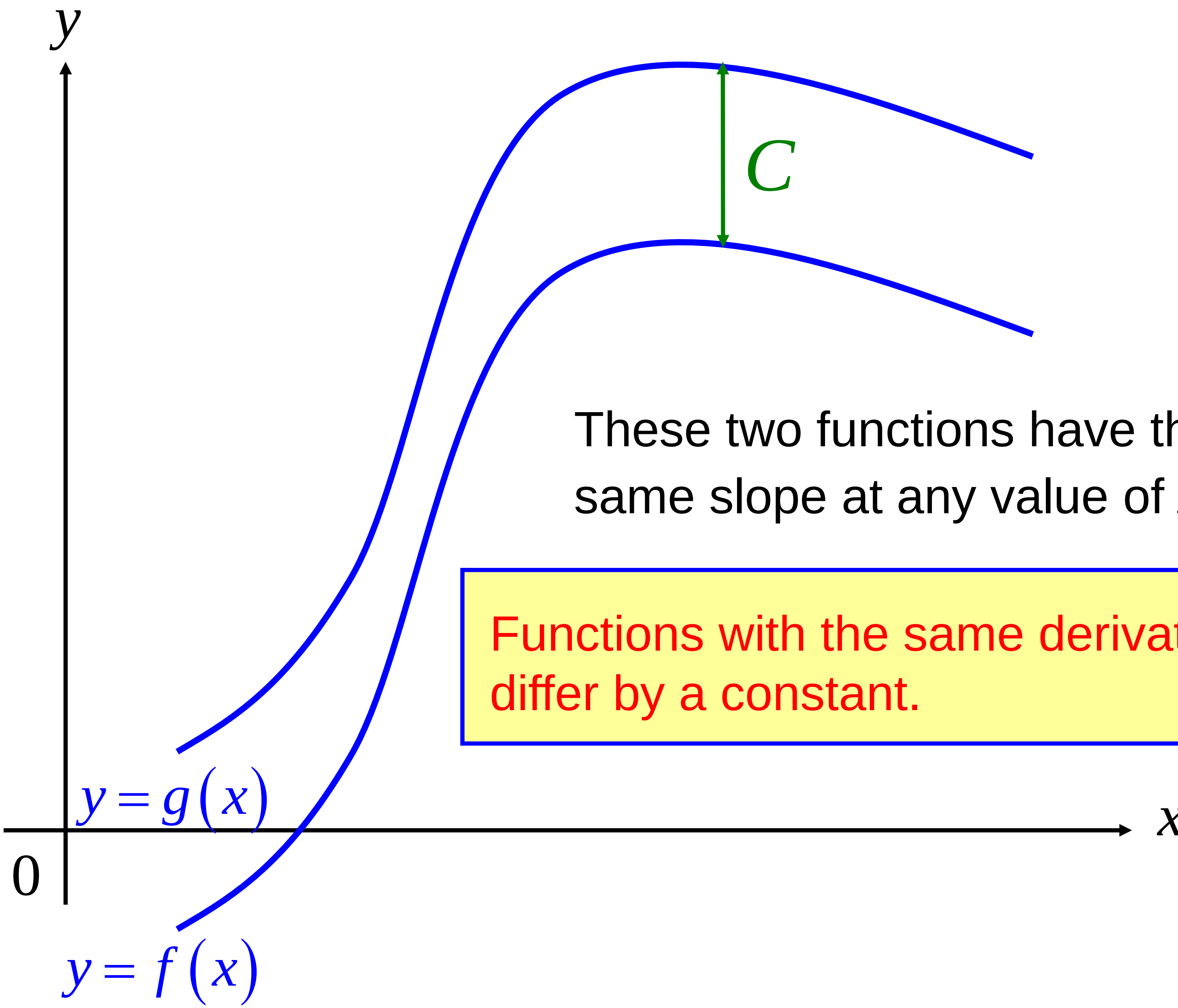


Tangent parallel
to chord.
Slope of tangent:
$f'(c)$
B
Slope of chord:
$\frac{f(b)-f(a)}{b-a}$
A
y
x
0
a
c
b
$y=f(x)$
→

A function is <u>increasing</u> over an interval if the derivative is always positive.

A function is <u>decreasing</u> over an interval if the derivative is always negative.

→



These two functions have the same slope at any value of x.

Functions with the same derivative differ by a constant.

Find the function $f(x)$ whose derivative is $\sin(x)$ and whose graph passes through $(0,2)$.

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

so: $$\frac{d}{dx}-\cos(x) = \sin(x)$$

$$\therefore f(x) = -\cos(x) + C$$

$$2 = -\cos(0) + C$$

Example 6:

Find the function $f(x)$ whose derivative is $\sin(x)$ and whose graph passes through $(0,2)$.

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

so: $$\frac{d}{dx}-\cos(x) = \sin(x)$$

$$\therefore f(x) = -\cos(x) + C$$

$$2 = -\cos(0) + C$$

$$2 = -1 + C$$

$$3 = C$$

Notice that we had to have initial values to determine the value of C.

$$f(x) = -\cos(x) + 3$$

→

The process of finding the original function from the derivative is so important that it has a name:

Antiderivative

A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The process of finding an antiderivative is **antidifferentiation**.

You will hear much more about antiderivatives in the future.

This section is just an introduction.

→

Example 7b: Find the velocity and position equations for a downward acceleration of 9.8 m/sec^2 and an initial velocity of 1 m/sec downward.

$$a(t) = 9.8$$

(We let down be positive.)

$$v(t) = 9.8t + C$$

$$1 = 9.8(0) + C$$

$$1 = C$$

$$v(t) = 9.8t + 1$$

Example 7b: Find the velocity and position equations for a downward acceleration of 9.8 m/sec^2 and an initial velocity of 1 m/sec downward.

$$a(t) = 9.8$$

$$v(t) = 9.8t + C$$

$$1 = 9.8(0) + C$$

$$1 = C$$

$$v(t) = 9.8t + 1$$

$$s(t) = \frac{9.8}{2}t^2 + t + C$$

The power rule in reverse: Increase the exponent by one and multiply by the reciprocal of the new exponent.

→

Example 7b: Find the velocity and position equations for a downward acceleration of 9.8 m/sec^2 and an initial velocity of 1 m/sec downward.

$$a(t)=9.8$$

$$v(t)=9.8t+C$$

$$1=9.8(0)+C$$

$$1=C$$

$$v(t)=9.8t+1$$

$$s(t)=\frac{9.8}{2}t^2+t+C$$

$$s(t)=4.9t^2+t+C$$

The initial position is zero at time zero.

$$0=4.9(0)^2+0+C$$

$$0=C$$

$$s(t)=4.9t^2+t$$

π

Assignment: p 202 Exercises # 1-8, 29 - 34