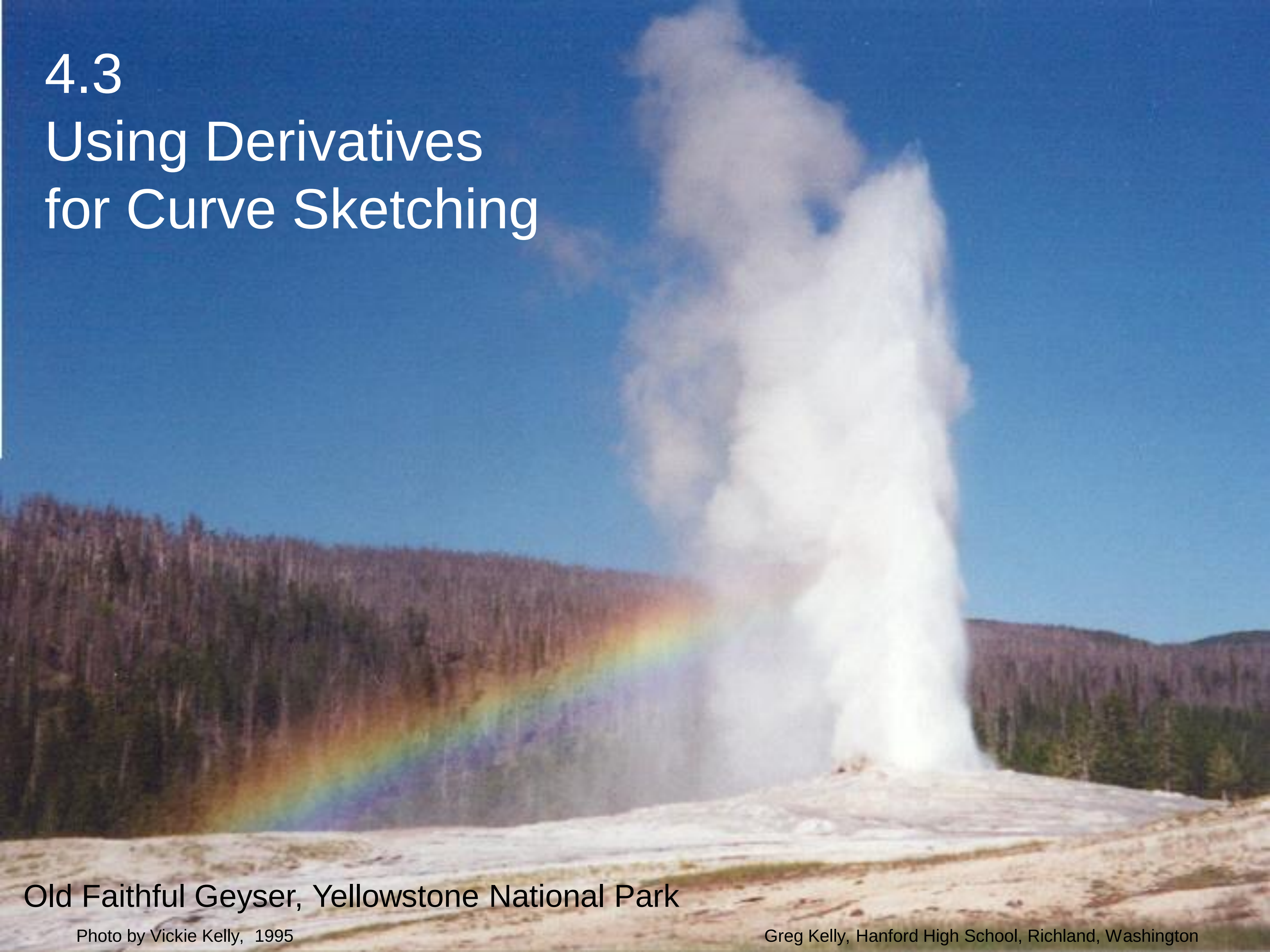
4.3
Using Derivatives for Curve Sketching



Old Faithful Geyser, Yellowstone National Park

Photo by Vickie Kelly, 1995

Greg Kelly, Hanford High School, Richland, Washington

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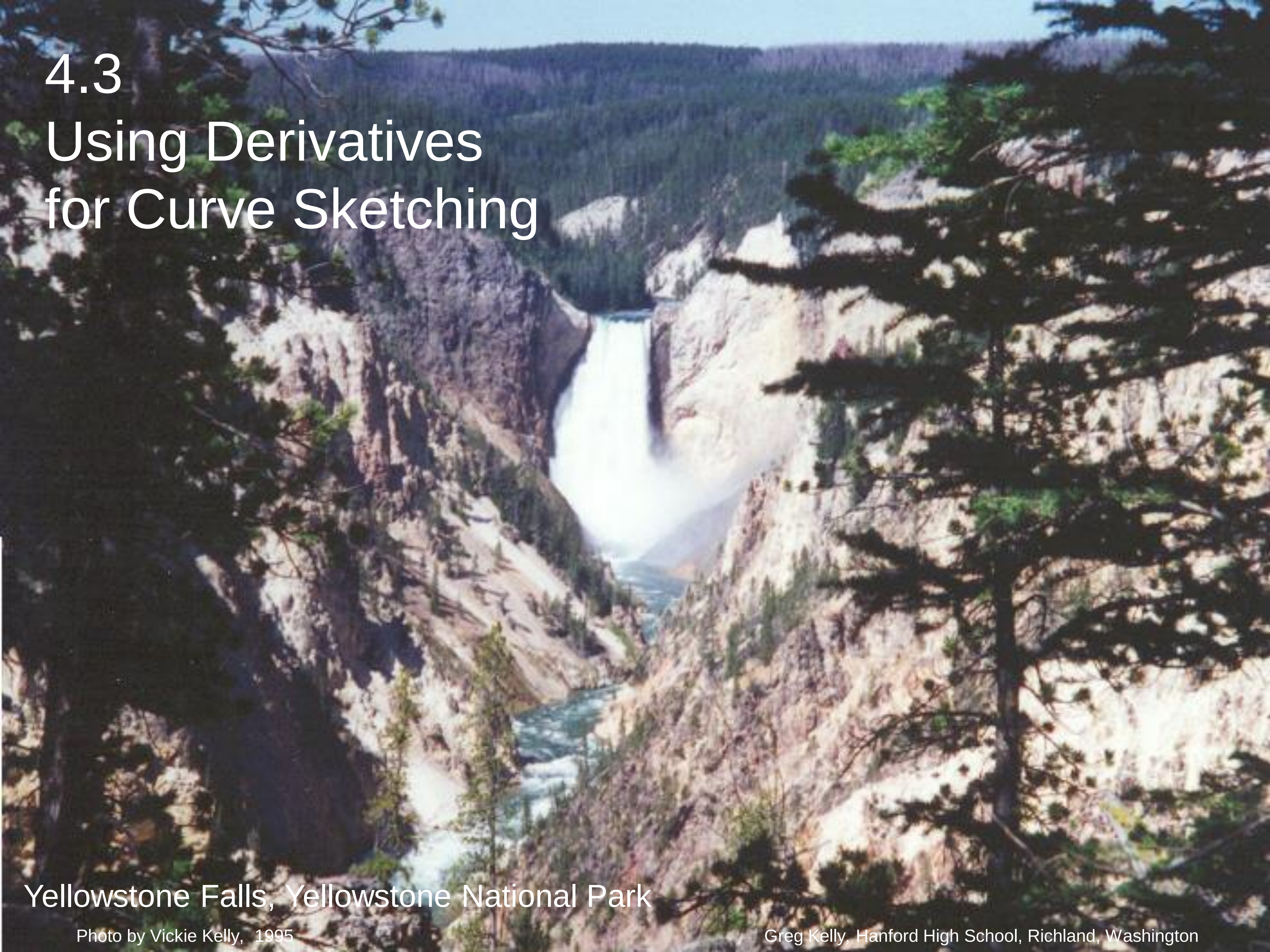


Yellowstone Falls, Yellowstone National Park

Photo by Vickie Kelly, 1995

Greg Kelly, Hanford High School, Richland, Washington

In the past, one of the important uses of derivatives was as an aid in curve sketching. We usually use a calculator of computer to draw complicated graphs, it is still important to understand the relationships between derivatives and graphs.

First derivative:

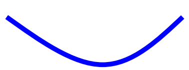
y' is positive → Curve is increasing.

y' is negative → Curve is decreasing.

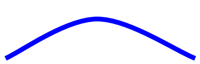
y' is zero → Possible local maximum or minimum.

Second derivative:

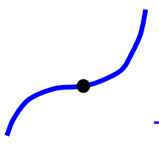
y'' is positive → Curve is concave up.



y'' is negative → Curve is concave down.



y'' is zero → Possible inflection point (where concavity changes).



Example:

Graph $y = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

There are roots at $x = -1$ and $x = 2$.

$y' = 3x^2 - 6x$

Possible extreme at $x = 0,\ 2$.

Set $y' = 0$

First derivative test:

$0 = 3x^2 - 6x$

y'	$+$	0	$-$	0	$+$
		0		2	

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0,\ 2$

$y'(1) = 3 \cdot 1^2 - 6 \cdot 1 = -3$ ➡ negative

$y'(-1) = 3(-1)^2 - 6(-1) = 9$ ➡ positive

$y'(3) = 3 \cdot 3^2 - 6 \cdot 3 = 9$ ➡ positive

$\rightarrow$

Example: **Graph** $y = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

There are roots at $x = -1$ and $x = 2$.

$y' = 3x^2 - 6x$

Set $y' = 0$

$0 = 3x^2 - 6x$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0,\ 2$

<u>Possible</u> extreme at $x = 0,\ 2$.

First derivative test:

y'	$+$	0	$-$	0	$+$
x		0		2	

$\therefore$ maximum at $x = 0$

minimum at $x = 2$

→

Example: **Graph** $y = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

There are roots at $x = -1$ and $x = 2$.

$y' = 3x^2 - 6x$

Possible extreme at $x = 0,\ 2$.

Set $y' = 0$

$0 = 3x^2 - 6x$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0,\ 2$

Or you could use the second derivative test:

$y'' = 6x - 6$

$y''(0) = 6 \cdot 0 - 6 = -6$ → negative

concave down

local maximum

$y''(2) = 6 \cdot 2 - 6 = 6$ → positive

concave up

local minimum

$\therefore$ maximum at $x = 0$ minimum at $x = 2$

→

Example:

Graph $y = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

We then look for inflection points by setting the second derivative equal to zero.

$$y'' = 6x - 6$$

$$0 = 6x - 6$$

$$6 = 6x$$

$$1 = x$$

Possible inflection point at $x = 1$.

	$-$	0	$+$
y''		1	

$y''(0) = 6 \cdot 0 - 6 = -6$ ➡ negative

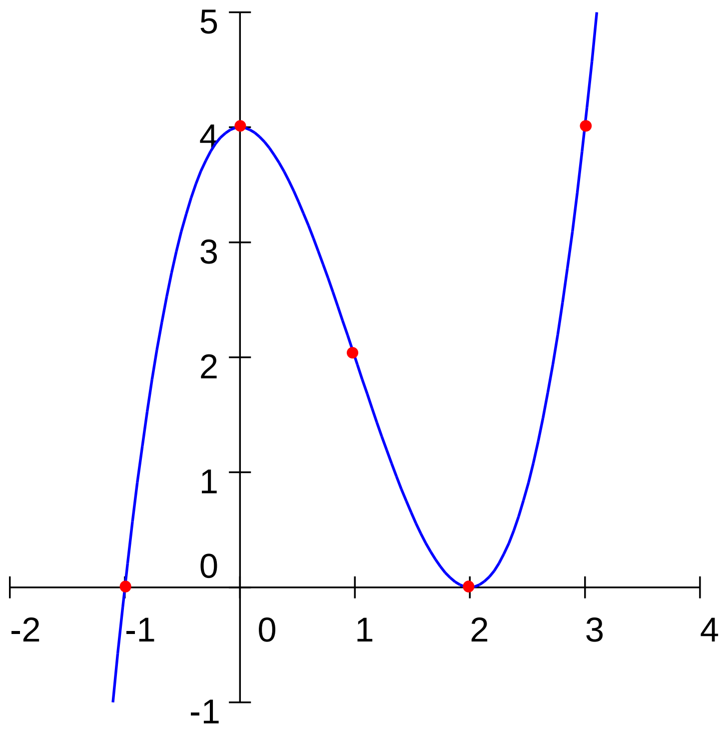
$y''(2) = 6 \cdot 2 - 6 = 6$ ➡ positive

$\therefore$ inflection point at $x = 1$

→

Make a summary table:

x	y	y'	y''	
-1	0	9	-12	rising, concave down
0	4	0	-6	local max
1	2	-3	0	falling, inflection point
2	0	0	6	local min
3	4	9	12	rising, concave up



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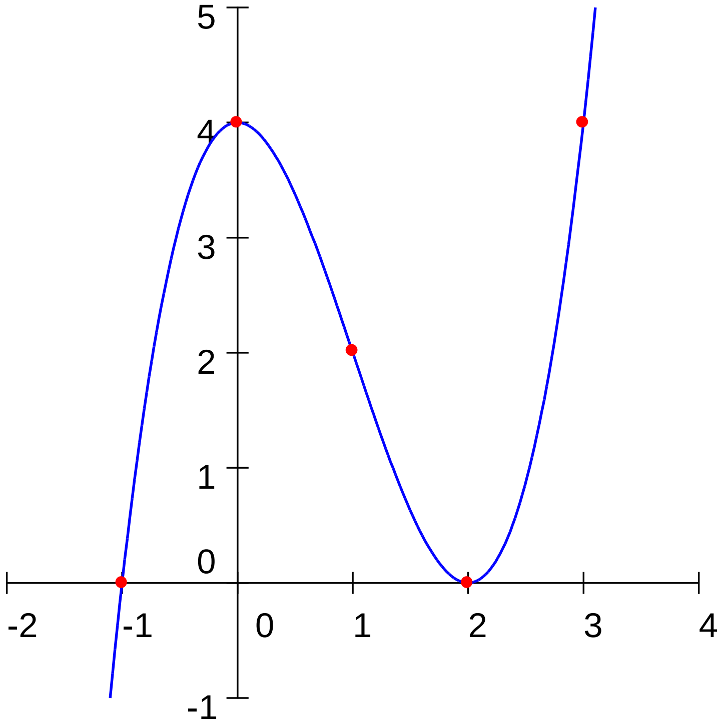

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