Warm-up

Suppose that position equation for a moving object is given by $s(t) = 10t^2 + t - 5$ where s is measured in meters and t is measured in seconds.

a) Find the velocity of the object when t=2.
b) Find the acceleration when t = 2.
c) Find the time of the object when the object when the velocity is 0.

4.6: Related Rates

Suppose that the radius is changing at an instantaneous rate of 0.1 cm/sec.

(Possible if the sphere is a soap bubble or a balloon.)

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi (10\text{cm})^2 \cdot \left(0.1 \frac{\text{cm}}{\text{sec}}\right)$$

$$\frac{dV}{dt} = 40\pi \frac{\text{cm}^3}{\text{sec}}$$

The sphere is growing at a rate of $40\pi \text{ cm}^3/\text{sec}$.

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Water is draining from a cylindrical tank at 3 liters/second. How fast is the surface dropping?

$$\frac{dV}{dt} = -3\frac{\text{L}}{\text{sec}} = -3000\frac{\text{cm}^3}{\text{sec}}$$

Find $\frac{dh}{dt}$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$-3000\frac{\text{cm}^3}{\text{sec}} = \pi r^2 \frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = -\frac{3000\frac{\text{cm}^3}{\text{sec}}}{\pi r^2}$$

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Steps for Related Rates Problems:

1. Draw a picture (sketch).
2. Write down known information.
3. Write down what you are looking for.
4. Write an equation to relate the variables.
5. Differentiate both sides with respect to t.
6. Evaluate.

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Hot Air Balloon Problem:

Given: $\theta = \frac{\pi}{4} \quad \frac{d\theta}{dt} = 0.14 \frac{\text{rad}}{\text{min}}$

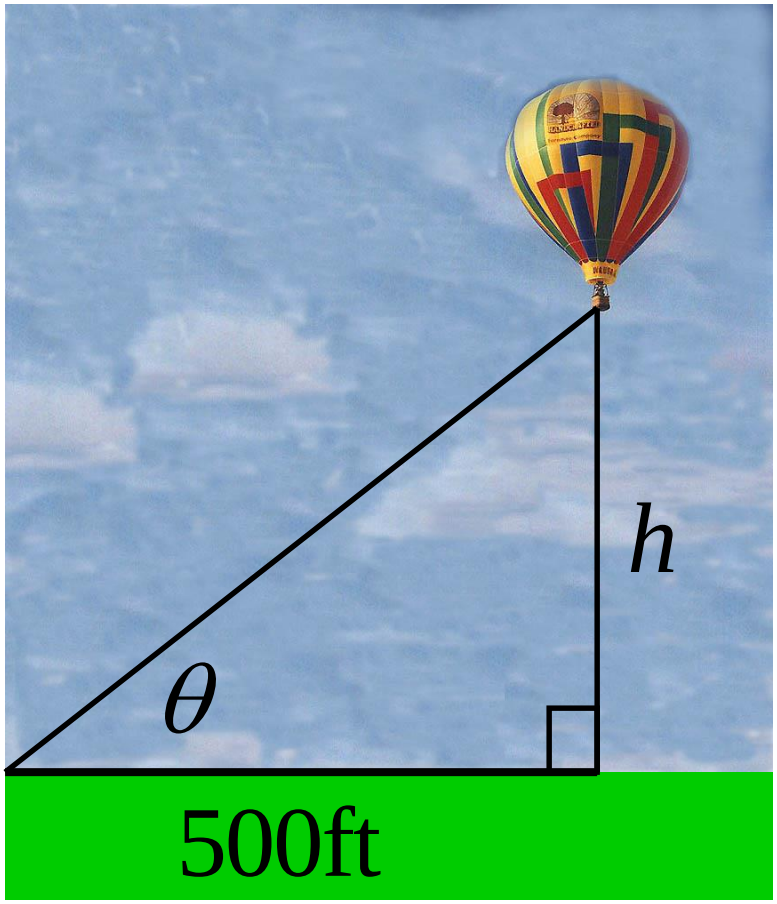
How fast is the balloon rising?

Find $\frac{dh}{dt}$

$$\tan\theta = \frac{h}{500}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{500}\frac{dh}{dt}$$

$$\left(\sec\frac{\pi}{4}\right)^2 (0.14) = \frac{1}{500}\frac{dh}{dt}$$



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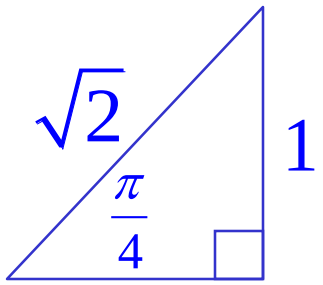

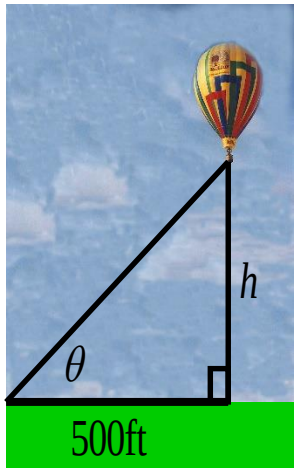

How fast is the balloon rising?

Find $\frac{dh}{dt}$

$$\sec\frac{\pi}{4} = \sqrt{2}$$

$$\tan\theta = \frac{h}{500}$$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{500}\frac{dh}{dt}$$

$$\left(\sqrt{2}\right)^2(0.14)\cdot 500 = \frac{dh}{dt}$$

$$\left(\sec\frac{\pi}{4}\right)^2(0.14) = \frac{1}{500}\frac{dh}{dt}$$

$$140\frac{\text{ft}}{\text{min}} = \frac{dh}{dt}$$

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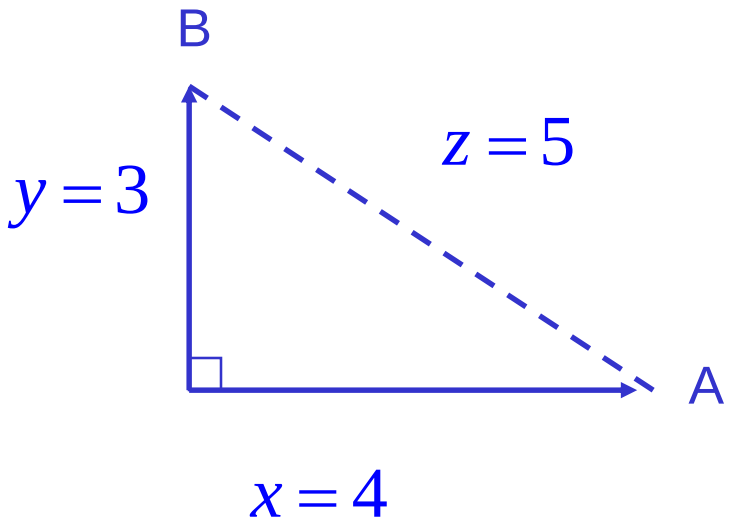
Truck Problem:

Truck A travels east at 40 mi/hr.
Truck B travels north at 30 mi/hr.

How fast is the distance between the trucks changing 6 minutes later?



$$r \cdot t = d$$

$$40 \cdot \frac{1}{10} = 4 \qquad 30 \cdot \frac{1}{10} = 3$$

$$3^2 + 4^2 = z^2$$

$$9 + 16 = z^2$$

$$25 = z^2$$

$$5 = z$$

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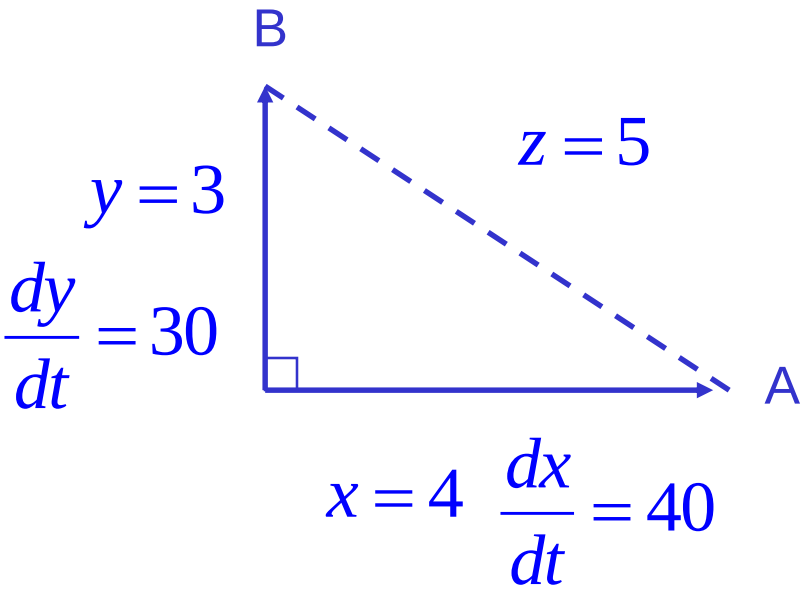

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$4 \cdot 40 + 3 \cdot 30 = 5\frac{dz}{dt}$$

$$250 = 5\frac{dz}{dt} \qquad 50 = \frac{dz}{dt}$$

$$50 \frac{\text{miles}}{\text{hour}}$$

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