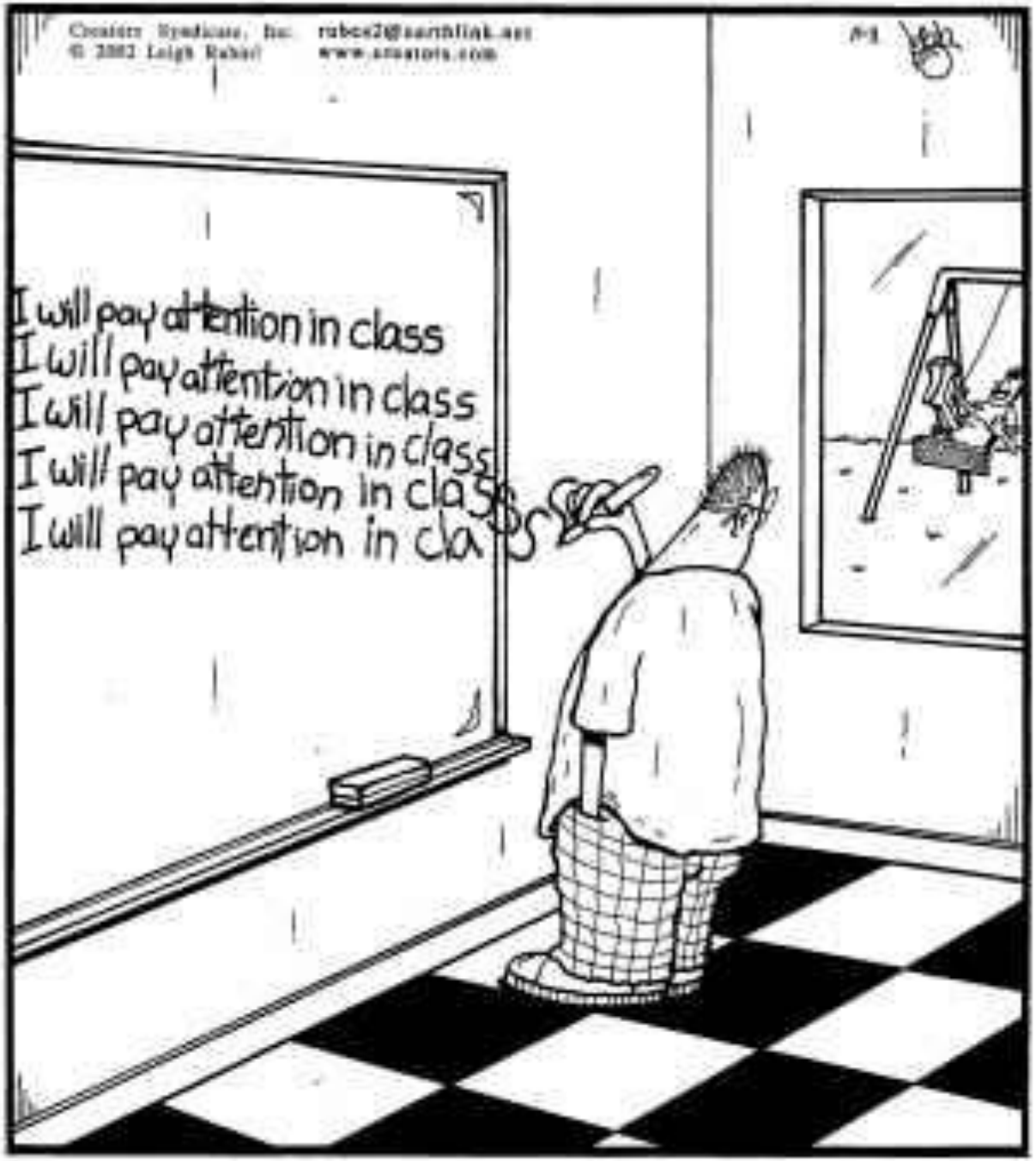
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3.6 The Chain Rule

Remember the composition of two functions?

$$f \circ g = f(g(x))$$

The chain rule is used when you have the composition of two functions.

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Find y' for $y = (x^2 + 1)^3$

$f(x) = x^3$

$g(x) = x^2 + 1$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

$$y' = 3(x^2 + 1)^2\,(2x) \quad = \quad 6x(x^2 + 1)^2$$

Find y' for $y = (3x - 2x^2)^3$

$$y' = 3(3x - 2x^2)^2(3 - 4x)$$

Find f'(x) for $f(x) = \sqrt[3]{(x^2+2)^2} = \left(x^2+2\right)^{2/3}$

$$f'(x) = \frac{2}{3}\left(x^2+2\right)^{-1/3}(2x) \quad = \frac{4x}{3\sqrt[3]{x^2+2}}$$

Differentiate

$g(t)=\dfrac{-7}{(2t-3)^2}$ **rewritten as** **= -7(2t – 3)$^{-2}$**

g’(t) = 14(2t – 3)$^{-3}$(2) $=\dfrac{28}{(2t-3)^3}$

Differentiate

$f(x)=x^2\sqrt{1-x^2}$ **rewritten as** $x^2\left(1-x^2\right)^{1/2}$

$$f'(x)=x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)+\left(1-x^2\right)^{1/2}(2x)$$

$$=-x^3\left(1-x^2\right)^{-1/2}+2x\left(1-x^2\right)^{1/2}$$

Factor

$$= x\left(1-x^2\right)^{-1/2}\left[-x^2+2\left(1-x^2\right)\right] = \frac{x\left(2-3x^2\right)}{\sqrt{1-x^2}}$$

Differentiate

$$f(x) = \frac{x}{\sqrt[3]{x^2+4}} \quad \textbf{rewritten as} \quad = \frac{x}{\left(x^2+4\right)^{1/3}}$$

Quotient Rule $\dfrac{\textbf{Bot * Top' – Top * Bot'}}{\textbf{(Bot)}^2}$

$$f'(x) = \frac{(x^2+4)^{1/3}(1) - x(1/3)(x^2+4)^{-2/3}(2x)}{(x^2+4)^{2/3}} \cdot \left(\frac{3}{3}\right)$$

$$= \frac{3(x^2+4)^{1/3} - 2x^2(x^2+4)^{-2/3}}{3(x^2+4)^{2/3}} \qquad = \frac{x^2+12}{3\left(x^2+4\right)^{4/3}}$$

Differentiate

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

$$y'=2\left(\frac{3x-1}{x^2+3}\right)^1\left(\frac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)^2}\right)$$

$$=\frac{2(3x-1)(3x^2+9-6x^2+2x)}{(x^2+3)^3}$$

$$=\frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx} \qquad \frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx} \qquad \frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}[\cot u] = -\csc^2 u \frac{du}{dx} \qquad \frac{d}{dx}[\csc u] = -\csc u \cot u \frac{du}{dx}$$

Applying the Chain Rule to trigonometric functions

$y = \sin 2x$ $\quad y' = (\cos 2x)\ (2) = 2 \cos 2x$

$y = \cos (x - 1)$ $\quad y' = -\sin (x - 1)\ (1) = -\sin (x - 1)$

$y = \tan 3x$ $\quad y' = \sec^2 3x\ (3) = 3 \sec^2 3x$

$y = \cos (3x^2)$ $\quad y' = -\sin (3x^2)\ (6x) = -6x \sin (3x^2)$

$y = \cos^2 3x$ $\quad$ rewritten as $\quad y = (\cos 3x)^2$

$y' = 2(\cos 3x)^1\ (-\sin 3x)\ (3)$

$y' = -6 \cos 3x \sin 3x$

Differentiate

$f(t) = \sqrt{\sin 4t}$ **rewritten as** $(\sin 4t)^{1/2}$

$$f'(t) = \frac{1}{2}(\sin 4t)^{-1/2} \cos 4t \ (4)$$

$$= \frac{2\cos 4t}{\sqrt{\sin 4t}}$$