

Warm-up

Homework problem #3 - rows 2, 4 and 6

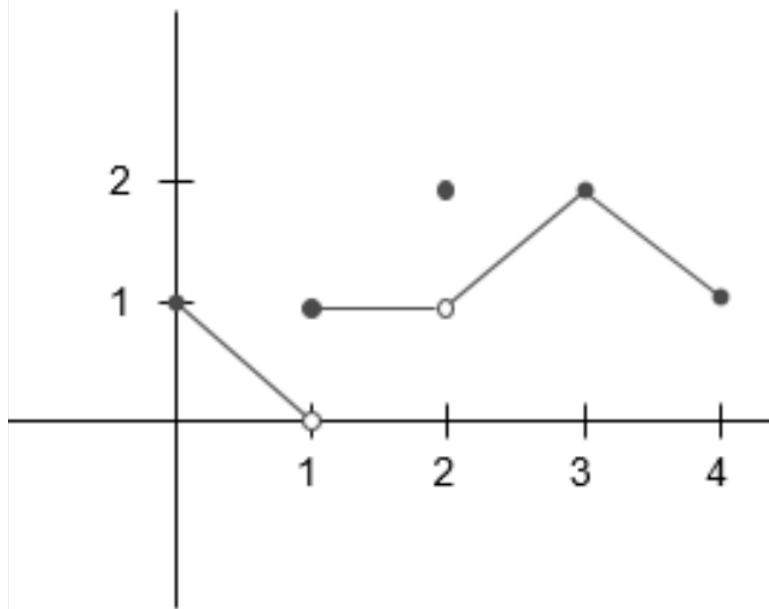
1.4: Continuity and IVT

I can:

Define and explore properties of continuity and use the Intermediate Value Theorem to prove continuity.

Most of the techniques of calculus require that functions be continuous. A function is continuous if you can draw it in one motion without picking up your pencil.

A function is continuous at a point if the limit is the same as the value of the function.



Types of Discontinuity

Removable Discontinuities:

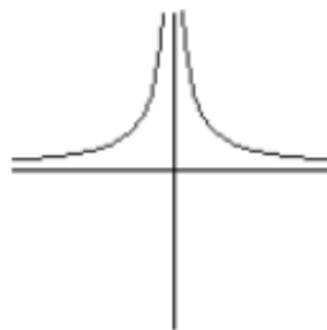


(You can fill the hole.)

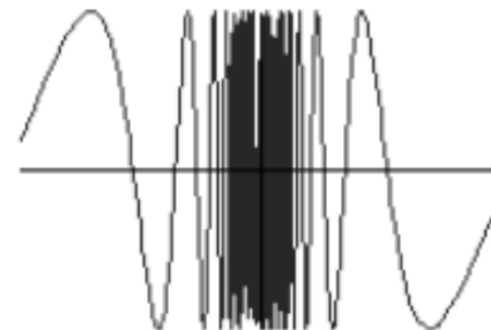
Essential Discontinuities:



jump



infinite



oscillating

Ex. 1 Is there a value of k that will make the following piecewise function continuous at $x = 3$?

$$f(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ 5-x, & 3 < x \leq 5 \end{cases}$$

2 What value of k will make $f(x)$ continuous?

$$f(x) = \begin{cases} x^2 - 2x - 3, & x \neq 2 \\ k - 3, & x = 2 \end{cases}$$

try! What value of k will make $f(x)$ continuous?

$$f(x) = \begin{cases} 4x - 11, & x < 3 \\ kx^2, & x \geq 3 \end{cases}$$

$$f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2 \end{cases}$$

Removing a discontinuity:

$$f(x) = \frac{x^3 - 1}{x^2 - 1} \quad \text{has a discontinuity at } x = 1 .$$

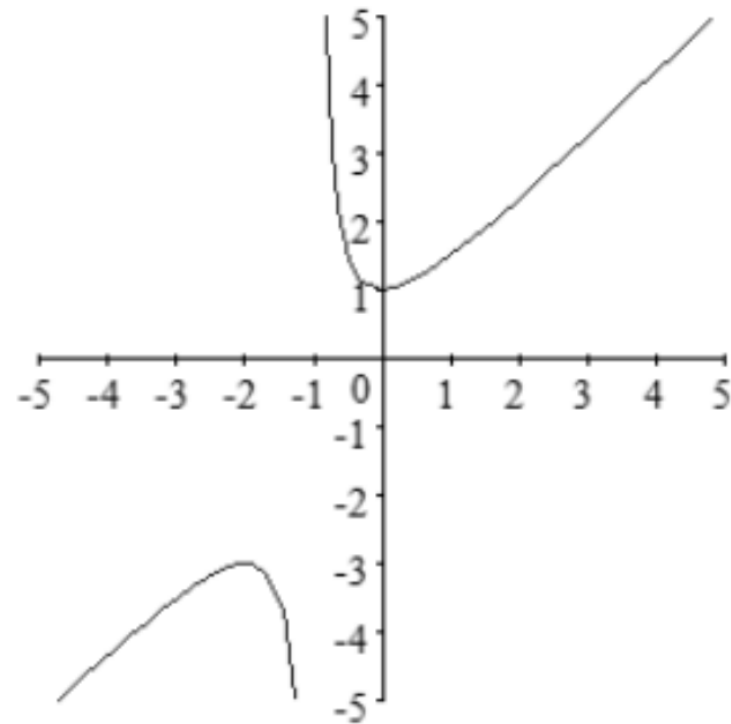
Write an extended function that is continuous at $x = 1$.

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{(x+1)\cancel{(x-1)}} = \frac{1+1+1}{2} = \frac{3}{2}$$

$$f(x) = \begin{cases} \frac{x^3 - 1}{x^2 - 1}, & x \neq 1 \\ \frac{3}{2}, & x = 1 \end{cases}$$

Note: There is another discontinuity at $x = -1$ that can not be removed.

Removing a discontinuity:

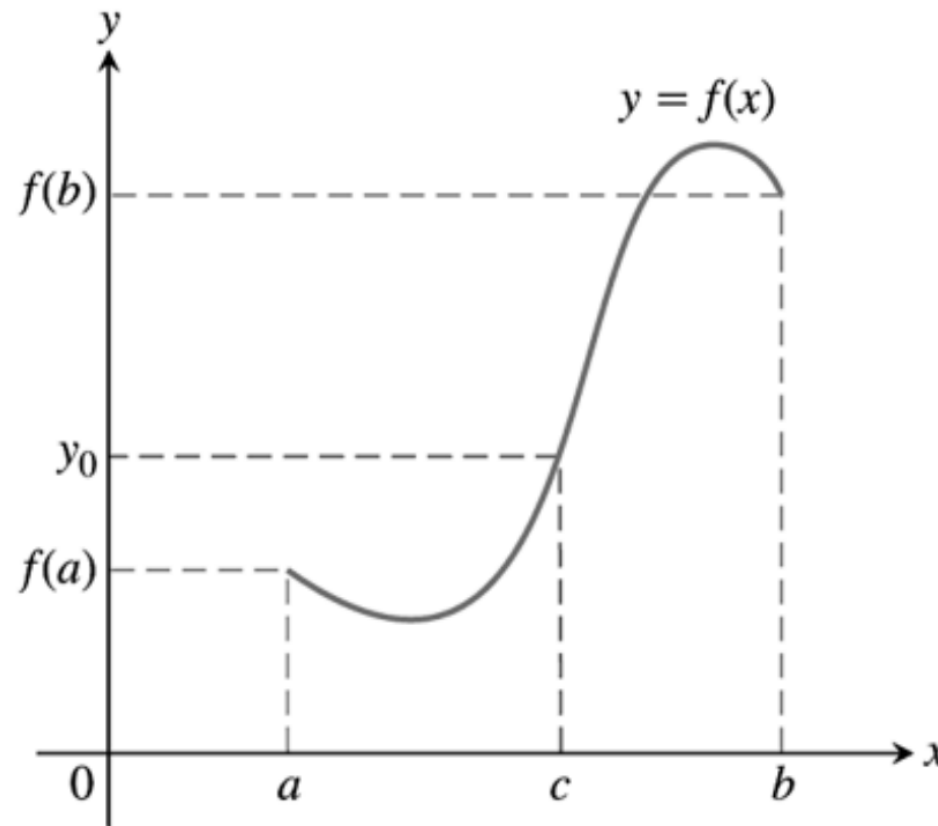


$$f(x) = \begin{cases} \frac{x^3 - 1}{x^2 - 1}, & x \neq 1 \\ \frac{3}{2}, & x = 1 \end{cases}$$

Note: There is another discontinuity at $x = -1$ that can not be removed.

Intermediate Value Theorem

function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



Examples

- If between 7am and 2pm the temperature went from 55 to 70.
 - At some time it reached 62.
 - Time is continuous
- If between his 14th and 15th birthday, a boy went from 150 to 165 lbs.
 - At some point he weighed 155lbs.
 - It may have occurred more than once.

3 Show that there exists a value c such that $f(c) = 2$ for the function $f(x) = x^2 + 2x - 3$ in the interval $[0, 2]$

4 Determine if any roots exist in the function

$$f(x) = x^2 - \sqrt{x+1} \quad \text{on the interval } [1,2]$$

y! Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero in the interval $[0, 1]$.

Directions:

1. Find person with same Greek letter.
2. Complete your assigned problem and show all work on separate sheet of paper.
3. Explain your thought process in two to four sentences.
4. Be ready to show work on board and explain.



