UNIT 7:

LOGARITHMIC FUNCTIONS

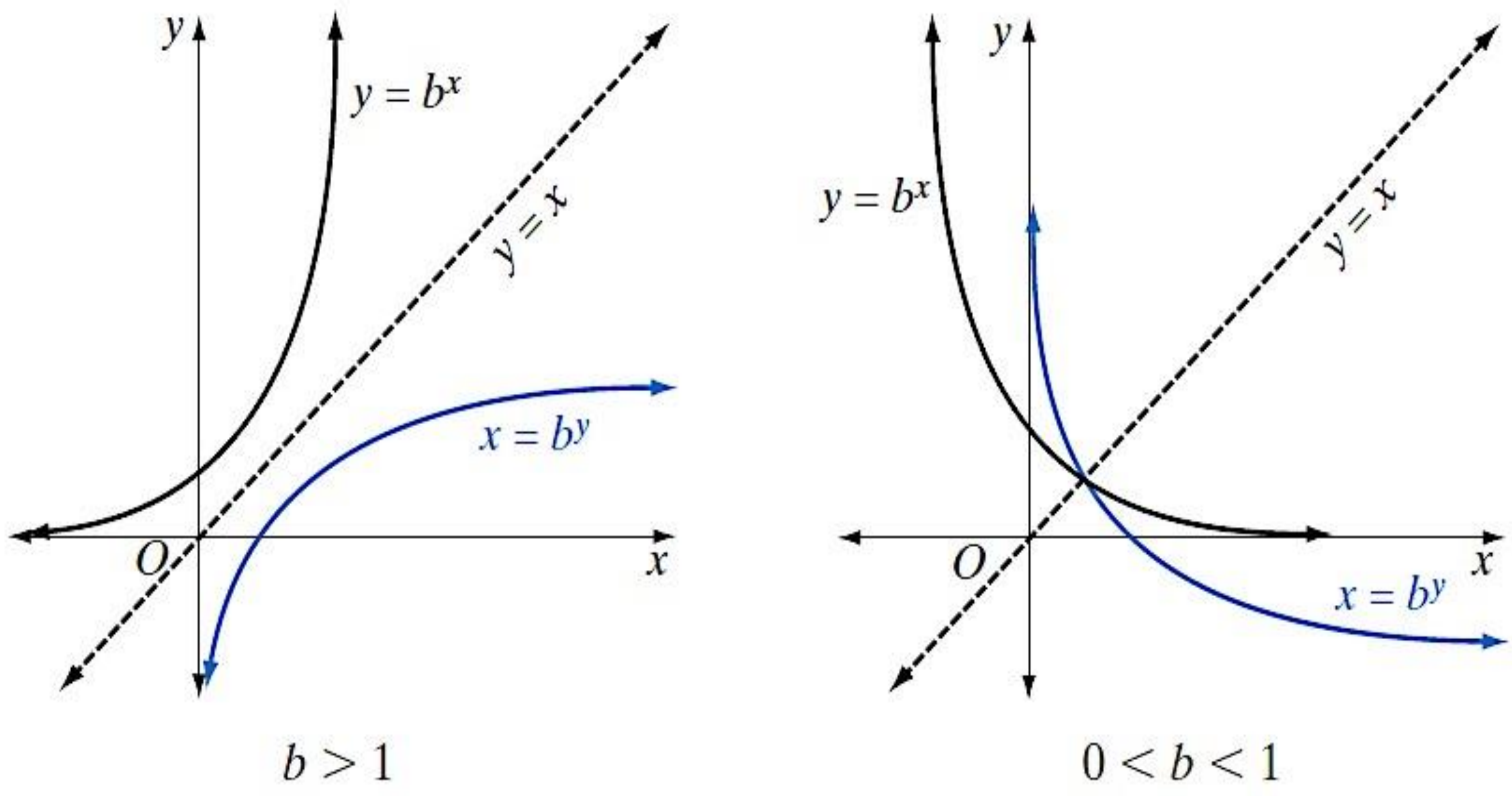
Exponential Function

- Where ***b*** is the **base** and *x* is the **exponent** (or **power**).
- If ***b*** is greater than 1, the function continuously increases in value as ***x*** increases.
- A special property of exponential functions is that the **slope** of the function also continuously increases as *x* increases.

Logarithmic Function

- Simply a way to express the **value of an exponent**.
- The **base** of the logarithm is ***b***.
- The logarithmic function has many real-life applications, in acoustics, electronics, earthquakes analysis and population prediction

The Graph of the Exponential Function & the Inverse Function:



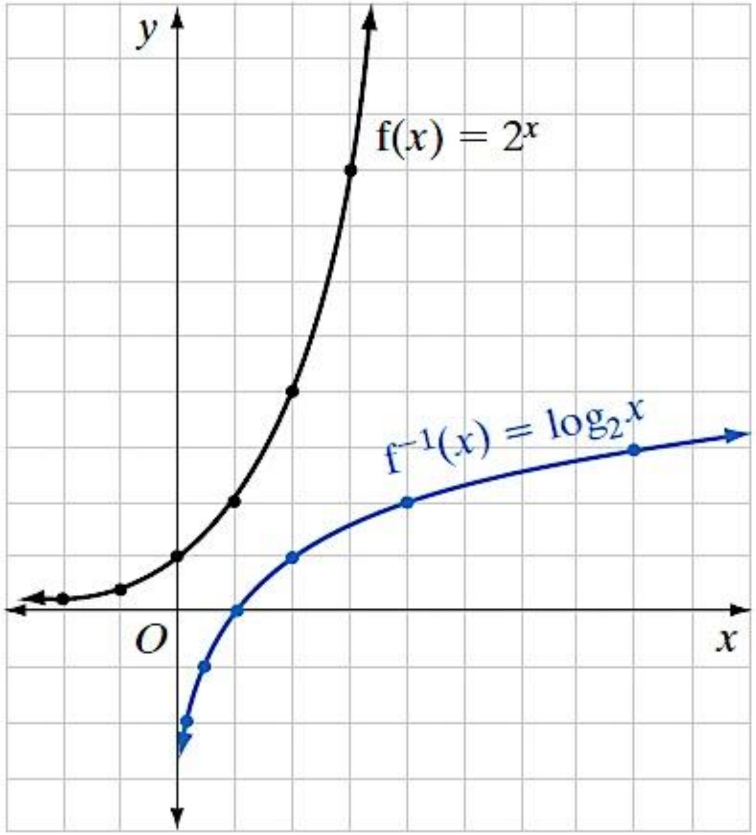
The Graph of the Exponential Function & The Inverse Function:

a. Sketch the graph of $f(x) = 2^x$.

b. Write the equation of $f^{-1}(x)$ and sketch its graph.

Solution **a.** Make a table of values for $f(x) = 2^x$, plot the points, and draw the curve.

x	2^x	f(x)
−2	$2^{-2} = \frac{1}{2^2}$	$\frac{1}{4}$
−1	$2^{-1} = \frac{1}{2}$	$\frac{1}{2}$
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8



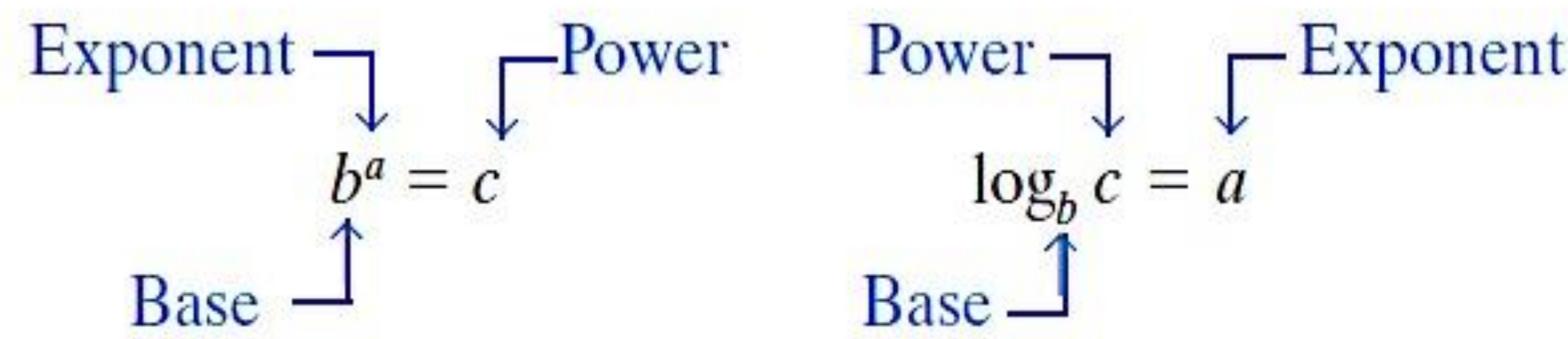
b. Let $f(x) = 2^x \rightarrow y = 2^x$.

To write $f^{-1}(x)$, interchange x and y.

$x = 2^y$ is written as $y = \log_2 x$. Therefore, $f^{-1}(x) = \log_2 x$.

Exponential Form & Logarithmic Form

- $b^a = c$ **is equivalent to** $\log_b c = a$.



Basic Properties of Logarithms

If $0 < b < 1$ or $b > 1$:

$$b^0 = 1 \leftrightarrow \log_b 1 = 0$$

$$b^1 = b \leftrightarrow \log_b b = 1$$

Examples: Write in logarithmic form

1. $8 = 2^3$

2. $27 = 3^3$

3. $4^5 = 1024$

4. $y = 2^x$

Examples: Write in exponential form

1. $\log_5 x = 2$

3. $\log_3(x+1) = 2$

2. $\log_6 216 = 3$

4. $\log_x 4 = y$

Product & Addition Property

$$\log_b cd = \log_b c + \log_b d$$

- **The log of a product is the sum of the logs of the factors of the product.**

Examples: Write as a single log expression or write as the sum of multiple log expressions.

1. $\log 2 + \log 6$

2. $\log 7 + \log x$

3. $\log 10$

4. $\log 2z$

Quotient & Subtraction Property

$$\log_b \frac{c}{d} = \log_b c - \log_b d$$

- The log of a quotient is the log of the dividend minus the log of the divisor.

Examples: Write as a single log expression or write as the difference of multiple log expressions.

1. $\log 9 - \log 3$

2. $\log 24 - \log x$

3. $\log \frac{12}{4}$

4. $\log \frac{5a}{b}$

Power & Coefficient Property

$$\log_b c^a = a \log_b c$$

- The log of a power is the exponent times the log of the base.

Important Rule!!

- When you have a number under the radical we can rewrite that as a fractional exponent and then follow the *Power & Coefficient Property.*

$$\log\sqrt{x} = \log x^{\frac{1}{2}} = \frac{1}{2}\log x$$

Examples: Write the log expression with a leading coefficient or with an exponent.

1. $\log 6^2$
2. $\log x^y$
3. $a \log 5$
4. $3 \log 8$
5. $\log \sqrt[3]{y^2}$

Summary

	Logarithms
Multiplication	$\log_b cd = \log_b c + \log_b d$
Division	$\log_b \frac{c}{d} = \log_b c - \log_b d$
Logarithm of a Power	$\log_b c^a = a \log_b c$
Logarithm of 1	$\log_b 1 = 0$
Logarithm of the Base	$\log_b b = 1$

Put It All Together

- These properties will most likely all be presented together in a question!
- They will be given with all having the same base
- You will be asked to expand each of the expressions using the logarithmic properties

OR

- You will be asked to write each expression as a single logarithm

Examples: Expand or condense the following logarithmic expressions

1. $\frac{1}{2}\log_{10} x + z\log_{10} 6$

2. $\log_3 a - 2\log_3 (b+1)$

3. $\log_2 a^4 b^5$

4. $\log_4 \frac{m^9}{n^2}$

Common Log

- The graphing calculators are written in base 10.

DEFINITION

A **common logarithm** is a logarithm to the base 10.

$$\log_{10} 100 = \log 100$$

Examples:

1. $\log_{10} 4$
2. $\log_{10} 6$
3. $log_{10} 3$
4. $\log 17$ ←

Remember, if you don't see a ***10*** for the base it is still a common base so it can be plugged right into the calculator as is.

Natural Logs

DEFINITION

A **natural logarithm** is a logarithm to the base *e*.

$$\ln e = 1$$

- All of the Log Properties also apply to natural logs
  - Multiplication & Addition
  - Division & Subtraction
  - Power & Coefficient

Examples: Simplify the following expressions

1. $\ln e^{x}$

3. $\ln 2 + \ln 3$

2. $\ln e^{-0.017}$

4. $\ln 7 - \ln 4$

Examples: Simplify the following expressions

5. $\ln 3^2$

7. $\ln 3^4 + \ln 4^5$

6. $5\ln 8$

8. $\ln x^2 - \ln y^3$

Examples: Solve the equation by using Natural Logs

9. $e^{x} = 3$

10. $e^{2x} = 34$

Homework:

- Complete all the ***even numbers*** from the work sheet provided