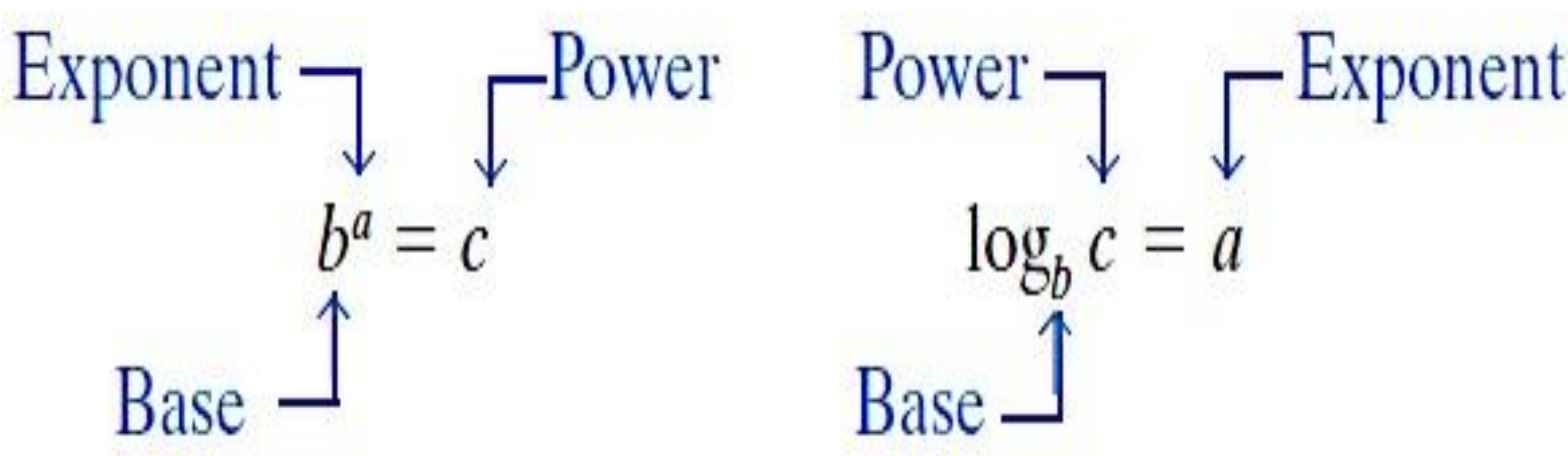
Unit7:
Evaluating Logarithmic & Exponential Equations

Exponential Form & Logarithmic Form

- $b^a = c$ is equivalent to $\log_b c = a$.



Property Summary:

	Powers	Logarithms
Exponent of Zero	$b^0 = 1$	$\log_b 1 = 0$
Exponent of One	$b^1 = b$	$\log_b b = 1$
Products	$b^{x+y} = cd$	$\log_b cd = \log_b c + \log_b d$ $= x + y$
Quotients	$b^{x-y} = \frac{c}{d}$	$\log_b \frac{c}{d} = \log_b c - \log_b d$ $= x - y$
Powers	$(b^x)^a = b^{ax} = c^a$	$\log_b c^a = a \log_b c$ $= ax$

*Evaluating Logarithmic Equations

- If $x_1 = x_2$, then $\log_b x_1 = \log_b x_2$.

- If $\log_b x_1 = \log_b x_2$, then $x_1 = x_2$.

*Change of Base Method

Solving Exponential Equations

For example, solve $8^x = 32$ for x. There are two possible methods.

METHOD I

Write each side of the equation to the base 2.

$$8^x = 32$$

$$(2^3)^x = 2^5$$

$$3x = 5$$

$$x = \frac{5}{3}$$

Check: $8^{\frac{5}{3}} = (\sqrt[3]{8})^5 = 2^5 = 32$ ✔

*Practice:

1. $6^{2x+12} = 6^{68}$

3. $27^{2x+3} = 3^{21}$

2. $64^{x+6} = 4^{x^2}$

4. $16^{x+10} = 2^{60}$

* Evaluating Logarithmic Equations:

For example, solve $8^x = 32$ for x. There are two possible methods.

METHOD 2

Take the log of each side of the equation and solve for the variable.

$$8^x = 32$$
$$\log 8^x = \log 32$$
$$x \log 8 = \log 32$$
$$x = \frac{\log 32}{\log 8}$$

ENTER: LOG 32) ÷ LOG 8 ENTER MATH ENTER ENTER

DISPLAY:

```
LOG(32)/LOG(8
      1.666666667
Ans►Frac
              5/3
```

Check: $8^{\frac{5}{3}} = (\sqrt[3]{8})^5 = 2^5 = 32$ ✔

*Practice:

1. $6^{2x+12} = 6^{68}$

3. $27^{2x+3} = 3^{21}$

2. $64^{x+6} = 4^{x^2}$

4. $16^{x+10} = 2^{60}$

*Practice:

*Using Ln to Solve Equations:

EXAMPLE 1

Solve for x to the nearest hundredth: $5.00(7.00)^x = 1{,}650$.

Solution *How to Proceed*

(1) Write the equation:

(2) Write the natural log of each side of the equation:

(3) Simplify the equation:

(4) Solve the equation for x:

(5) Use a calculator to compute x:

EXAMPLE 1

Solve for x to the nearest hundredth: $5.00(7.00)^x = 1{,}650$.

$$5.00(7.00)^x = 1{,}650$$

$$\ln 5.00(7.00)^x = \ln 1{,}650$$

$$\ln 5.00 + \ln 7.00^x = \ln 1{,}650$$

$$\ln 5.00 + x \ln 7.00 = \ln 1{,}650$$

$$x \ln 7 = \ln 1{,}650 - \ln 5.00$$

$$x = \frac{\ln 1{,}650 - \ln 5.00}{\ln 7.00}$$

ENTER: (LN 1650) − LN 5)) ÷ LN 7 ENTER

DISPLAY:

```
(LN(1650)-LN(5))
/LN(7
           2.980144102
```

Answer $x \approx 2.98$

* Evaluating with Ln on both sides:

EXAMPLE 1

Solve for x and check: $\ln 12 - \ln x = \ln 3$.

Solution *How to Proceed*

(1) Write the equation:

(2) Solve for $\ln x$:

(3) Simplify the right side of the equation:

(4) Equate the antilog of each side of the equation:

* Evaluating with Ln on both sides:

EXAMPLE 1

Solve for x and check: $\ln 12 - \ln x = \ln 3$.

$$\ln 12 - \ln x = \ln 3$$

$$-\ln x = -\ln 12 + \ln 3$$

$$\ln x = \ln 12 - \ln 3$$

$$\ln x = \ln \frac{12}{3}$$

$$x = \frac{12}{3}$$

$$x = 4$$

* Evaluating with Ln on both sides:

Alternative Solution

How to Proceed

(1) Write the equation:

(2) Simplify the left side of the equation:

(3) Equate the antilog of each side of the equation:

(4) Solve for x:

$$\ln 12 - \ln x = \ln 3$$

$$\ln \frac{12}{x} = \ln 3$$

$$\frac{12}{x} = 3$$

$$12 = 3x$$

$$4 = x$$

Check

$$\ln 12 - \ln x = \ln 3$$

$$\ln 12 - \ln 4 \stackrel{?}{=} \ln 3$$

$$\ln \frac{12}{4} \stackrel{?}{=} \ln 3$$

$$\ln 3 = \ln 3 \checkmark$$

*Solving by Calculator

Calculator check

ENTER: LN 12) − LN 4 ENTER

LN 3 ENTER

DISPLAY:

```
LN(12)-LN(4
          1.098612289
LN(3
          1.098612289
```

Answer $x = 4$

* Solving Logarithmic Equations for x

EXAMPLE 2

Solve for x: $\log x + \log (x + 5) = \log 6$.

Solution (1) Write the equation:

(2) Simplify the left side:

(3) Equate the antilog of each side of the equation:

(4) Solve the equation for x:

EXAMPLE 2

Solve for x: $\log x + \log (x + 5) = \log 6$.

$$\log x + \log (x + 5) = \log 6$$

$$\log [x(x + 5)] = \log 6$$

$$x(x + 5) = 6$$

Reject the negative root. In the given equation, $\log x$ is only defined for positive values of x.

$$x^2 + 5x = 6$$

$$x^2 + 5x - 6 = 0$$

$$(x - 1)(x + 6) = 0$$

$x - 1 = 0$	$x + 6 = 0$
$x = 1$	$x = -6$ ✘

Answer $x = 1$

EXAMPLE 2

Solve for x: $\log x + \log (x + 5) = \log 6$.

Check

$$\log x + \log (x + 5) = \log 6$$

$$\log 1 + \log (1 + 5) \stackrel{?}{=} \log 6$$

$$0 + \log 6 \stackrel{?}{=} \log 6$$

$$\log 6 = \log 6$$

EXAMPLE 3

Solve for b: $\log_b 8 = \log_4 64$.

Solution Let each side of the equation equal x.

$$\text{Let } x = \log_4 64.$$
$$4^x = 64$$
$$(2^2)^x = 2^6$$
$$2^{2x} = 2^6$$
$$2x = 6$$
$$x = 3$$

$$\text{Let } x = \log_b 8.$$
$$3 = \log_b 8$$
$$b^3 = 8$$
$$(b^3)^{\frac{1}{3}} = 8^{\frac{1}{3}}$$
$$b = \sqrt[3]{8}$$
$$b = 2$$

EXAMPLE 3

Solve for b: $\log_b 8 = \log_4 64$.

Solution Let each side of the equation equal x.

$$\text{Let } x = \log_4 64.$$
$$4^x = 64$$
$$(2^2)^x = 2^6$$
$$2^{2x} = 2^6$$
$$2x = 6$$
$$x = 3$$

$$\text{Let } x = \log_b 8.$$
$$3 = \log_b 8$$
$$b^3 = 8$$
$$(b^3)^{\frac{1}{3}} = 8^{\frac{1}{3}}$$
$$b = \sqrt[3]{8}$$
$$b = 2$$

Check

$$\log_b 8 = \log_4 64$$
$$\log_2 8 \stackrel{?}{=} \log_4 64$$
$$3 = 3 \checkmark$$

*Homework

* *Work sheet – Even numbers only
* *Add the following questions to your homework and solve by change of base method.

$2^{4x-5} = 32^{x}$ $\quad 5^{2x} = 125^{2x+2}$ $\quad 3^{3x+2} = 27^{3}$