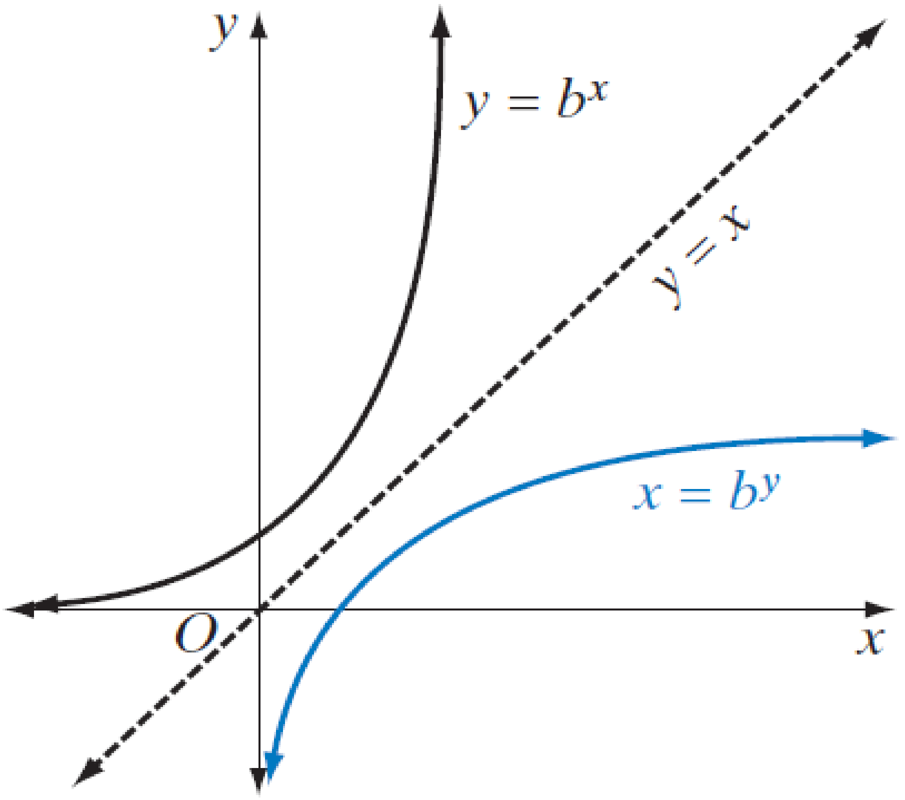
Unit 7:
Graph of Logarithmic Functions

Exponential Function & The Inverse Graph

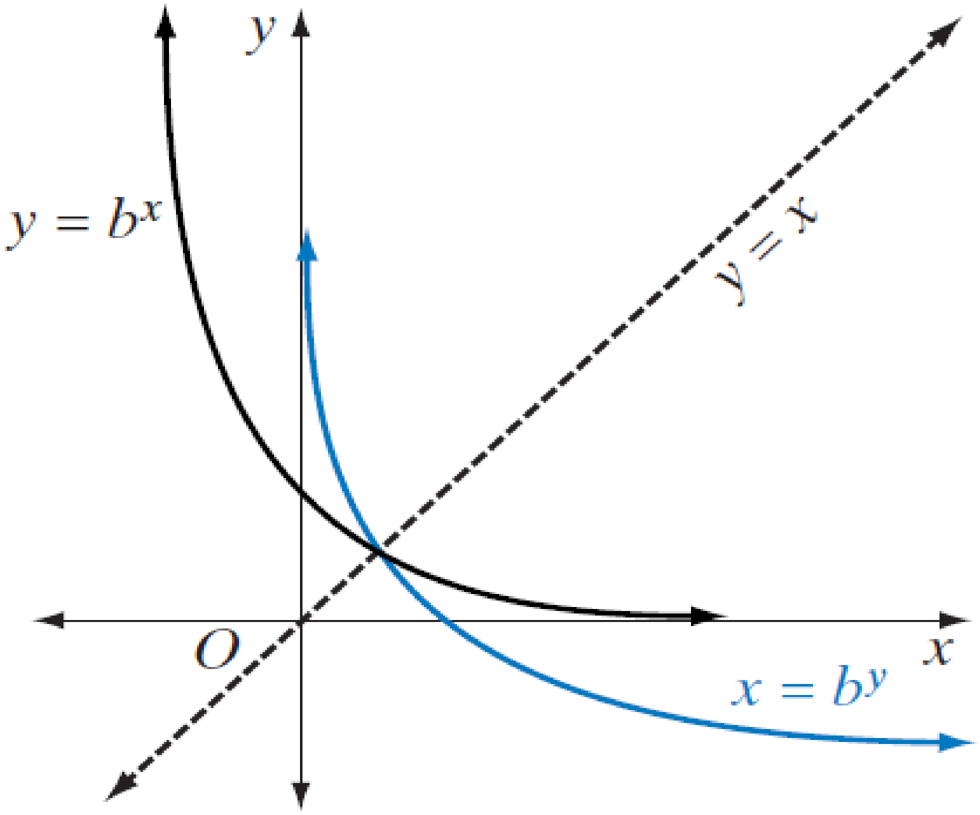
In Chapter 7, we showed that any positive real number can be the exponent of a power by drawing the graph of the exponential function $y = b^x$ for $0 < b < 1$ or $b > 1$. Since $y = b^x$ is a one-to-one function, its reflection in the line $y = x$ is also a function. The function $x = b^y$ is the inverse function of $y = b^x$.



$b > 1$

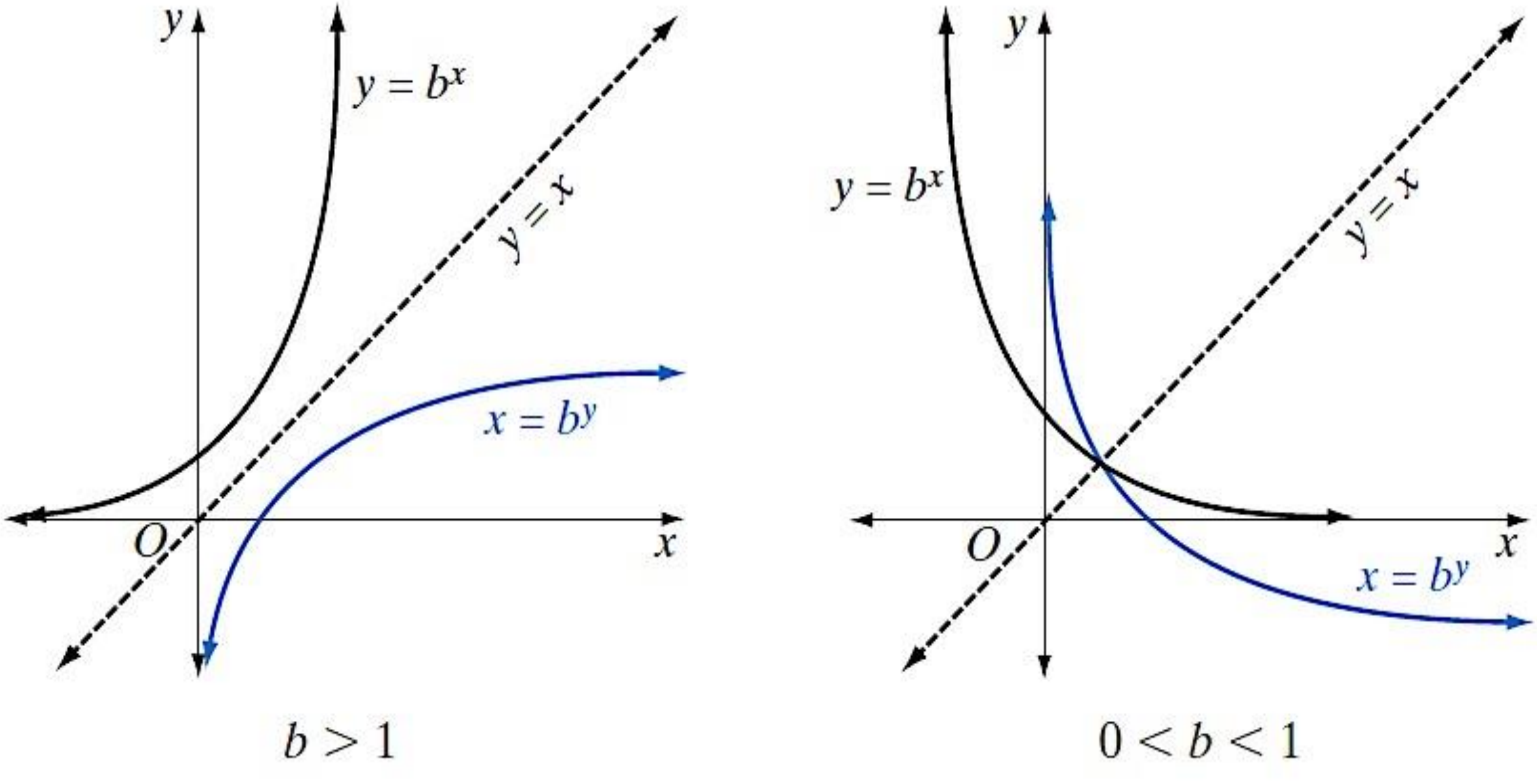
The equation of a function is usually solved for y in terms of x. To solve the equation $x = b^y$ for y, we need to introduce some new terminology. First we will describe y in words:

$x = b^y$: "y is the exponent to the base b such that the power is x."



$0 < b < 1$

The Graph of the Exponential Function & the Inverse Function:



Exponential Function & The Inverse

A **logarithm** is an exponent. Therefore, we can write:

$x = b^y$: "y is the *logarithm* to the base b of the power x."

The word *logarithm* is abbreviated as *log*. Look at the essential parts of this sentence:

$y = \log_b x$: "$\boldsymbol{y}$ **is** the **log**arithm to the **base** $\boldsymbol{b}$ of $\boldsymbol{x}$."

The base b is written as a subscript to the word "log."

▶ $\boldsymbol{x = b^y}$ **can be written as** $\boldsymbol{y = \log_b x}$**.**

For example, let $b = 2$. Write pairs of values for $x = 2^y$ and $y = \log_2 x$.

$x = 2^y$	In Words	$y = \log_2 x$	(x, y)
$\frac{1}{2} = 2^{-1}$	-1 is the logarithm to the base 2 of $\frac{1}{2}$.	$-1 = \log_2 \frac{1}{2}$	$\left(\frac{1}{2}, -1\right)$
$1 = 2^0$	0 is the logarithm to the base 2 of 1.	$0 = \log_2 1$	$(1, 0)$
$\sqrt{2} = 2^{\frac{1}{2}}$	$\frac{1}{2}$ is the logarithm to the base 2 of $\sqrt{2}$.	$\frac{1}{2} = \log_2 \sqrt{2}$	$\left(\sqrt{2}, \frac{1}{2}\right)$
$2 = 2^1$	1 is the logarithm to the base 2 of 2.	$1 = \log_2 2$	$(2, 1)$
$4 = 2^2$	2 is the logarithm to the base 2 of 4.	$2 = \log_2 4$	$(4, 2)$
$8 = 2^3$	3 is the logarithm to the base 2 of 8.	$3 = \log_2 8$	$(8, 3)$

We say that $y = \log_b x$, with b a positive number not equal to 1, is a **logarithmic function**.

EXAMPLE 1

Write the equation $x = 10^y$ for y in terms of x.

Solution $x = 10^y \leftarrow y$ is the exponent or logarithm to the base 10 of x.

$y = \log_{10} x$

When we interchange x and y to form the inverse function $x = b^y$ or $y = \log_b x$:

- **The domain of $y = \log_b x$ is the set of positive real numbers.**
- **The range $y = \log_b x$ is the set of real numbers.**
- **The y-axis or the line $x = 0$ is a *vertical asymptote* of $y = \log_b x$.**

EXAMPLE 2

a. Sketch the graph of $f(x) = 2^x$.

b. Write the equation of $f^{-1}(x)$ and sketch its graph.

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Solution **a.** Make a table of values for $f(x) = 2^x$, plot the points, and draw the curve.

x	2^x	f(x)
-2	$2^{-2} = \frac{1}{2^2}$	$\frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$	$\frac{1}{2}$
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8

EXAMPLE 2

a. Sketch the graph of $f(x) = 2^x$.

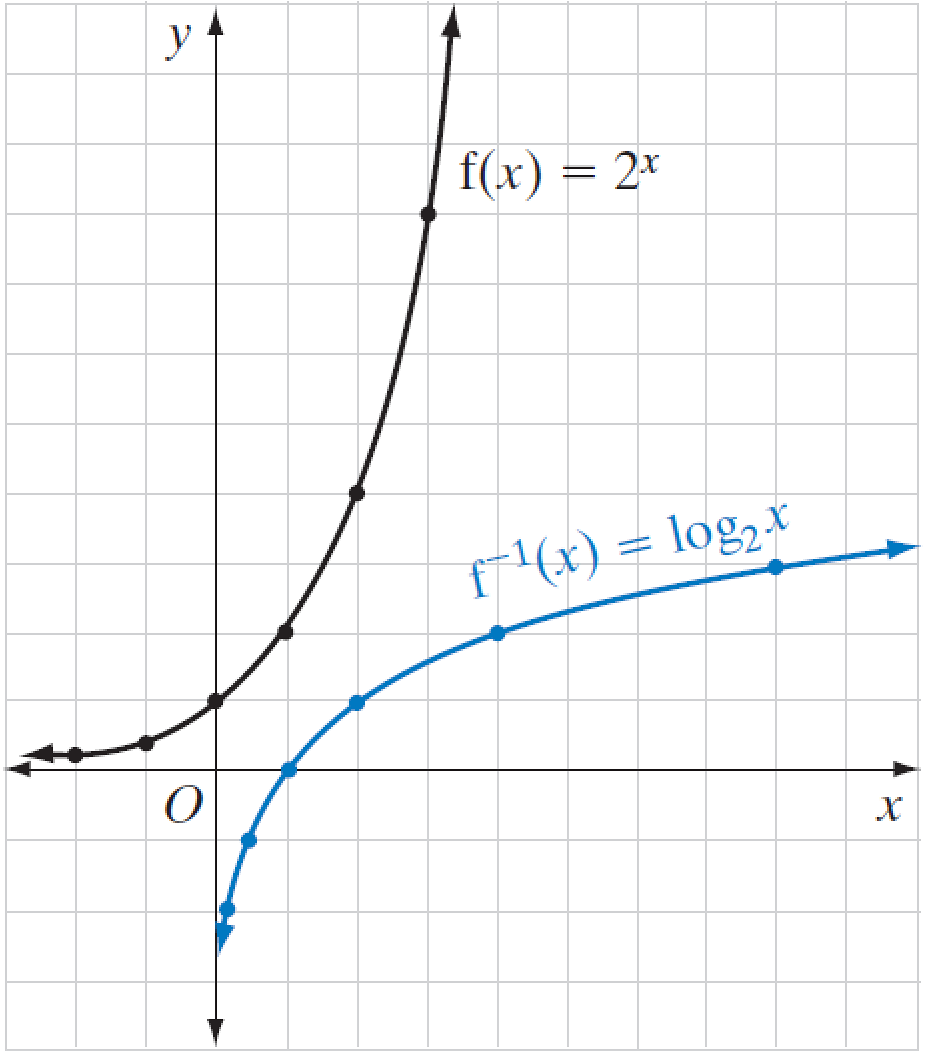
b. Write the equation of $f^{-1}(x)$ and sketch its graph.

b. Let $f(x) = 2^x \rightarrow y = 2^x$.

To write $f^{-1}(x)$, interchange x and y.

$x = 2^y$ is written as $y = \log_2 x$. Therefore, $f^{-1}(x) = \log_2 x$.

To draw the graph, interchange x and y in each ordered pair or reflect the graph of $f(x)$ over the line $y = x$. Ordered pairs of $f^{-1}(x)$ include $\left(\frac{1}{4}, -2\right)$, $\left(\frac{1}{2}, -1\right)$, $(1, 0)$, $(2, 1)$, $(4, 2)$, and $(8, 3)$.



Homework

- Problems from the given work sheet