

State the pattern for each set.

- $\textcircled{+3}$ 3, 6, 9, 12, 15, 18, 21, 24, ...

-5 -5

- 25, 20, 15, 10, 5, 0, -5, -10, ...

$$3 + (-5) = -2$$
$$-2 + (-5) = -7$$

- $\textcircled{+4}$ -21, 3, 27, 51, 75, 99, 123, ...

+24

Unit 8: Recursive Functions

8.1

ARITHMETIC SEQUENCES

Definition: Arithmetic Sequence

A sequence in which there is a **common difference (d)** between consecutive terms.

$$-26, -21, -16, -11, -6, \dots$$

+5 *+5* *+5* **d = 5**

Is the given sequence arithmetic? If so, identify the common difference.

2, 4, 8, 16, ... no

4, 6, 12, 18, 24, ... no

2, 5, 7, 12, ... no

48, 45, 42, 39, ... yes $d = -3$

1, 4, 9, 16, ... no

10, 20, 30, 40, ... yes $d = 10$

Arithmetic Sequence Formula

The 1st term in the sequence.

The common difference.

$$a_n = a_1 + (n - 1) \cdot d$$

The “nth” term in the sequence.

ex. a_5 is the 5th term.

The same as the n in a_n .

If you're looking for the 5th term in the sequence, $n = 5$.

Example 1:

$$a_n = a_1 + (n - 1) \cdot d$$

Given the sequence $-4, 5, 14, 23, 32, 41, 50, \dots$,
find the 14^{th} term.

$$a_1 = -4$$

$$n = 14$$

$$d = 9$$

$$a_n = 113$$

$$a_n = -4 + (14 - 1) \cdot 9$$

$$a_n = 113$$

Example 2:

$$a_n = a_1 + (n - 1) \cdot d$$

Given the sequence 6, 10, 14, 18, 22, 26, ..., find the 17th term.

$$\begin{aligned} a_1 &= 6 \\ n &= 17 \\ d &= 4 \\ a_n &= a_{17} = 70 \end{aligned}$$

$$\begin{aligned} a_{17} &= 6 + (17-1)4 \\ a_{17} &= 6 + (16) \cdot 4 \\ a_{17} &= 70 \end{aligned}$$

PEMDAS

Example 3:

$$a_n = a_1 + (n - 1) \cdot d$$

Given the sequence 81, 80.5, 80, 79.5, 79, ..., find the 9th term.

$$\begin{aligned} a_1 &= 81 \\ n &= 9 \\ d &= -0.5 \end{aligned}$$

$$\begin{aligned} a_9 &= 81 + (9-1) \cdot (-0.5) \\ a_9 &= 77 \end{aligned}$$

Example 4:

$$a_n = a_1 + (n - 1) \cdot d$$

Given the sequence 79, 75, 71, 67, 63, ..., find the term that has a value of -169.

$$\begin{aligned} a_1 &= 79 \\ n &= ? \\ d &= -4 \\ a_n &= -169 \end{aligned}$$

$$n = 63$$

$$\begin{aligned} -169 &= 79 + (n-1)(-4) \\ -79 &= (n-1)(-4) \\ -248 &= (n-1)(-4) \\ \frac{-248}{-4} &= \frac{(n-1)(-4)}{-4} \end{aligned}$$

$$62 = n - 1$$

$$63 = n$$

Example 5:

$$a_n = a_1 + (n - 1) \cdot d$$

Given the sequence 4, 7, 10, 13, ..., find the term that has a value of 301.

$a_1 = 4$
 $n =$
 $d = 3$
 $a_n = 301$

$$301 = 4 + (n-1)(3)$$

$-4 \quad -4$

$$\frac{297}{3} = \frac{(n-1)(3)}{3}$$

$99 = n-1$
 $+1 \quad +1$

$= 100$

Try this!

Write a formula for the following sequences.

a) 1, 7, 13, 19, ...

b) The first term is 3 and the common difference is -21

c) The second term is 8 and the common difference is 3

d) $x+10$, $x+7$, $x+4$, $x+1$, ...

**Determine if the sequence is arithmetic. If it is, find the common difference.
Give the formula for each sequence.**

1) 5, 32, 29, 26, ...

Find a_{15}

2) 34, -64, -94, -124, ...

Find the 34th term

3) 7, -9, -11, -13, ...

Find the term that has the value of -91

4) -3, -23, -43, -63, ...

Find a_{10}

5) -30, -40, -50, -60, ...

Find the 21st term

6) 9, 14, 19, 24, ...

Find the term that has the value of 59

Recursive Formula for Arithmetic Sequences

- Each term in an arithmetic sequence can be obtained recursively from its preceding term by adding d :

$$a_n = a_{n-1} + d \text{ (for all } n \geq 2)$$

