# LINEAR FUNCTIONS

As previously described, a linear equation can be defined as an equation in which the highest exponent of the equation variable is one. A linear function is a function of the form $f(x) = ax + b$. The graph of a linear equation is a graphic view of the set of all points that make the equation true. The graph of any linear function is a straight line.

A linear function can be represented in two ways, standard form and slope-intercept form. Standard form is a formal way of writing a linear equation, while slope-intercept form makes the equation easier to graph.

Form	Equation	Note
Standard	$Ax + By = C$	A and B are not 0. $A > 0$
Slope-intercept	$y = mx + b$	m is the slope of the line and b is the y-intercept.

To graph a linear function we must first identify the x and y intercepts. The x-intercept is the point where the graph crosses the x-axis and the y-intercept is the point where the graph crosses the y-axis.
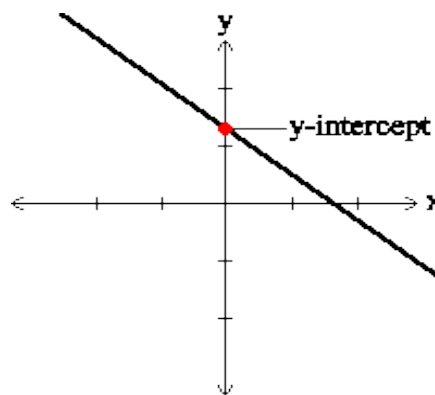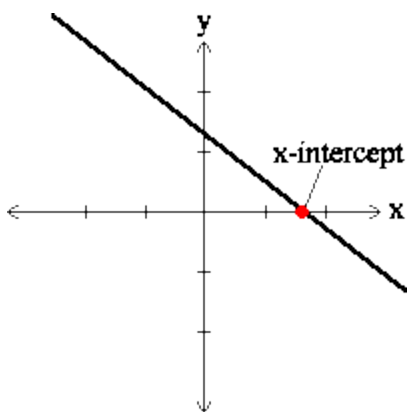

To find the x-intercept:

1. Set $y = 0$ in the equation.
2. Solve for x. The value obtained is the x-coordinate of the x-intercept.
3. The x-intercept is the point $(x, 0)$, with x the value found in step 2.

To find the y-intercept:

1. Set $x = 0$ in the equation.
2. Solve for y. The value obtained is the y-coordinate of the y-intercept.
3. The y-intercept is the point $(0, y)$, with y the value found in step 2.
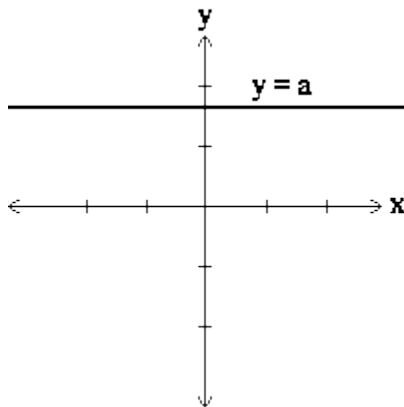
Vertical Lines

Equations of the form $x = a$ are vertical lines. The x-coordinate of every point on the vertical line $x = a$ has the value "a," or any given number.
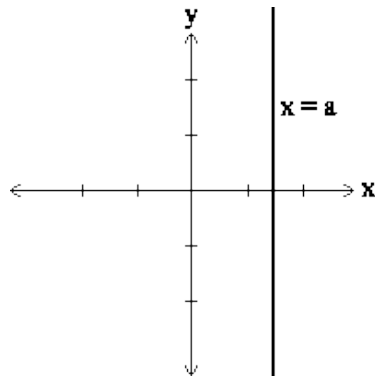
Horizontal Lines

Equations of the form $y = a$ are horizontal lines. The y-coordinate of every point on the horizontal line $y = a$ has the value "a," or any given number.
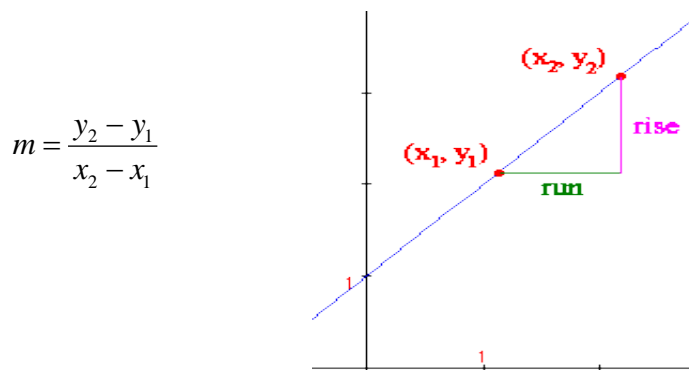
Slope

The slope of a line refers to the slant or inclination of the line. The slope is the ratio of the vertical change to the horizontal change between two points on the line. The slope can also be called the rise over run ratio because it tells how many spaces to move up or down and how many spaces to move to the right. A positive sign will move the line up and a negative sign will move the line down. One important thing to remember is that the run will always be to the right, regardless of the sign.

$$m = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta x}{\Delta y}$$

The formula to find the slope of a line passing through the points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: A horizontal line has slope of 0, while a vertical line has an undefined slope.
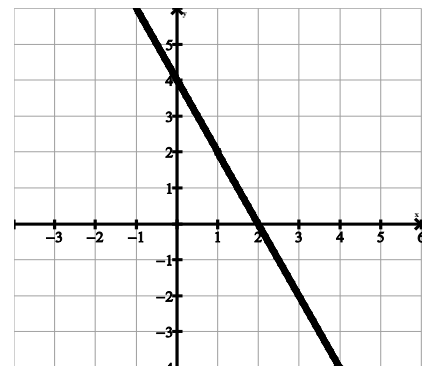
Example:

Find the slope of the line:

Use any two points on a line to calculate its slope. Your answer will be the same no matter which points you choose.

Choosing the points (1, 2) and (2, 0):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{2 - 1} = \frac{-2}{1} = -2$$

Parallel Lines

In the y-intercept form equation, $y = mx + b$, m is the slope and b is the y-intercept. Two lines are parallel if their slopes are the same ($m_1 = m_2$) and their y-intercepts are different ($b_1 \neq b_2$).

Example: 1) $y = 2x + 9$ $m_1 = 2$ $b_1 = 9$

                2) $y = 2x - 1$ $m_2 = 2$ $b_2 = -1$

In the previous example the slope of both equations is the same and their y-intercepts are different. Therefore, the lines are parallel.
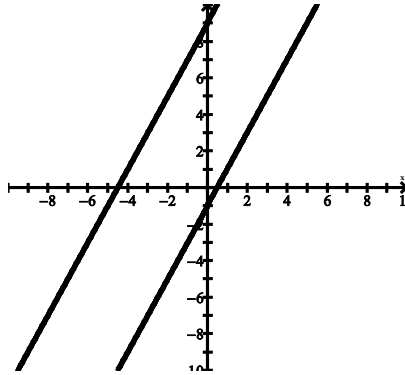
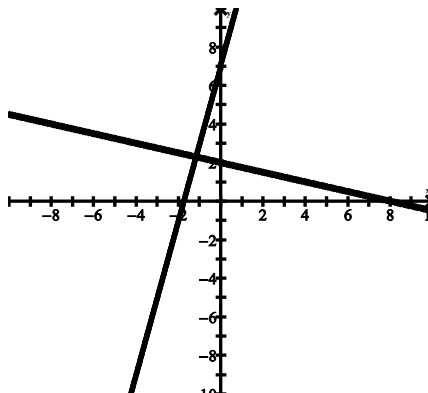**Perpendicular Lines**

Two lines are perpendicular if the slopes are the negative reciprocal of each other: $m_1 = -\dfrac{1}{m_2}$.

Example: 1) $y = 4x + 7$ $m_1 = 4$ $b_1 = 7$

                2) $y = -\dfrac{1}{4}x + 2$ $m_2 = -\dfrac{1}{4}$ $b_2 = 2$

The slope of equation 2 is the negative reciprocal of the slope of equation 1. Therefore, the lines are perpendicular.

Finding the Equation of a Line

To find the equation of a line when only a set of points or a slope is given, use the point-slope form of a linear equation formula:

$$y - y_1 = m(x - x_1)$$

where m is the slope of the line and (x_1, y_1) is a point on the line.

Examples:

Find the equation of the line which passes through the point (–4, 2) and whose slope is 5.

Using the point-slope form equation we obtain:

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 5(x - (-4))$$
$$y - 2 = 5x + 20$$
$$y = 5x + 22 \qquad \leftarrow Resultant\ Equation$$

Find the equation of the line through the points (–2, 5) and (–6, 4).

First find the slope between these two points using the slope equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 5}{-6 - (-2)} = \frac{-1}{-4} = \frac{1}{4}$$

With the slope obtained and one of the two points given, use the point-slope form equation to find the equation of the line.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{1}{4}(x - (-2))$$
$$y - 5 = \frac{1}{4}x + \frac{1}{2}$$
$$y = \frac{1}{4}x + \frac{11}{2} \qquad \leftarrow Resultant\ Equation$$

Eventually, you may be asked to find the equation of a line that is parallel or perpendicular to a given line. Remember that the slopes of two parallel lines are exactly the same and that the slopes of two perpendicular lines are the negative reciprocals of each other.

Examples:

Find the equation of a line that is parallel to $y = 4x + 2$ and that passes through the point (2, 7).

$$y = 4x + 2 \qquad \leftarrow \text{Since the equation is in slope-intercept form, the coefficient of } x \text{ is}$$
$$m_1 = 4 \qquad\qquad \text{the slope of the line. Remember y=mx+b (m is the slope!).}$$

Since the lines must be parallel, use the same slope and the given point to find the equation of the parallel line: $\qquad m_1 = m_2 = 4$

$$y - y_1 = m(x - x_1)$$
$$y - 7 = 4(x - 2)$$
$$y - 7 = 4x - 8$$
$$y = 4x - 1 \qquad \leftarrow \textit{Parallel line to } y = 4x + 2.$$

Find the equation of a line perpendicular to $4x + 2y = 9$ and that passes through the point (−1, −1).

$$4x + 2y = 9 \qquad\qquad \leftarrow \text{This equation is in standard form. We need to convert it}$$
$$\qquad\qquad\qquad\qquad \text{to slope-intercept form: y=mx+b.}$$
$$2y = -4x + 9$$
$$y = \frac{-4x + 9}{2}$$
$$y = -2x + \frac{9}{2} \qquad\qquad \leftarrow \text{The slope is } m_1 = -2.$$

Since the second equation must be perpendicular to the first equation, find the negative reciprocal.

$$m_2 = -\frac{1}{m_1} \qquad \rightarrow \qquad m_2 = -\frac{1}{-2} = \frac{1}{2}$$

Using the slope $m_2 = \dfrac{1}{2}$ and the point (−1, −1):

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{1}{2}(x - (-1))$$

$$y + 1 = \frac{1}{2}x + \frac{1}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2} \qquad \leftarrow \textit{Perpendicular line to } 4x + 2x = 9.$$

LINEAR FUNCTIONS – EXERCISES

1. Identify the false statement below
   a) The standard form of a linear equation is $y = ax + b$
   b) The point (4, 0) is on the graph of $3x - 4y = 12$
   c) The graph of $3x - 4y = 12$ passes the vertical line test for functions
   d) The y-intercept of $3x - 4y = 12$ is (0, –3)

2. Identify the false statement below
   a) To find the x-intercept of a graph, set y equal to zero and solve the equation for x
   b) The graph of $x = 3$ is a horizontal line three units above the x-axis
   c) Generally speaking, it is a good idea to plot three points when constructing the graph of a linear function
   d) To find the y-intercept of a graph, set x equal to zero and solve the equation for y

3. The standard form of the equation $5y - 6x = 30$ is

   a) $-6x + 5y = 30$ b) $y = \dfrac{6}{5}x + 6$

   c) $5y - 6x = 30$ d) $6x - 5y = -30$

4. Identify the true statement below
   a) All linear graphs are functions
   b) The graph of $y = 4$ is a function
   c) To find the y-intercept of a function, let y equal to zero and solve for x
   d) The graph of $x = 4$ is a function

5. Determine the slope and y-intercept of each equation:
   a) $6x + 7y = 14$ b) $2x + 5 = 0$ c) $3y + 9 = 0$

6. Use the point slope form to find the equation of the line which passes through (2, –1) and whose slope is $\dfrac{3}{7}$.

7. Write in standard form the equation of the line passing through the point (–3, –4) with slope equal to $m = \dfrac{3}{2}$.

8. Which of the following is the equation of a line in standard form passing through the point (3, 0) and perpendicular to the line $y - 2x = -1$?
   a) $2x + y = 6$ b) $x + 2y = 3$
   c) $2x - y = 6$ d) $x - 2y = 3$

9. Find the equation of a line in standard form that passes through the points (–1, 4) and (3, 2).

10. Determine whether the two lines are parallel, perpendicular or either: $3x - 5y = 6$ and $6x - 10y = 7$

LINEAR FUNCTIONS – ANSWERS TO EXERCISES

1. A is the false statement. The standard form of a linear equation is $Ax + By = C$.

2. B is the false statement. The graph of $x = 3$ is a vertical line 3 units to the right of the y-axis.

3. D: $6x - 5y = -30,\ A > 0$

4. B: $y = 4$ is an example of a linear function.

5. a) $m = -\dfrac{6}{7}$ and $b = 2$

   b) $m = undefinded$ and b does not exist

   c) $m = 0$ and $b = -3$

6. $y = \dfrac{3}{7}x - \dfrac{13}{7}$

7. $3x - 2y = -1$

8. B: $x + 2y = 3$ is a perpendicular line to $y - 2x = -1$

9. $x + 2y = 7$

10. The lines are parallel.