Exponents – Basic Definitions and Properties

For *any* real number **base** x, we define **powers** of x: $x^0 = 1$, $x^1 = x$, $x^2 = x \cdot x$, $x^3 = x \cdot x \cdot x$, etc. (The exception is 0^0, which is considered **indeterminate**.) Powers are also called **exponents**.

Examples: $5^0 = 1$, $(-11.2)^1 = -11.2$, $(8.6)^2 = 8.6 \times 8.6 = 73.96$, $10^3 = 10 \times 10 \times 10 = 1000$, $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81$.

Also, we can define *fractional* exponents in terms of **roots**, such as $x^{1/2} = \sqrt{x}$, the square root of x. Similarly, $x^{1/3} = \sqrt[3]{x}$, the cube root of x, $x^{2/3} = \left(\sqrt[3]{x}\right)^2$, etc. In general, we have $\boxed{x^{m/n} = \left(\sqrt[n]{x}\right)^m}$, i.e., the n^{th} root of x, raised to the m^{th} power.

Examples: $64^{1/2} = \sqrt{64} = 8$, $64^{3/2} = \left(\sqrt{64}\right)^3 = 8^3 = 512$, $64^{1/3} = \sqrt[3]{64} = 4$, $64^{2/3} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$.

Finally, we can define *negative* exponents: $\boxed{x^{-r} = \frac{1}{x^r}}$. Thus, $x^{-1} = \frac{1}{x^1}$, $x^{-2} = \frac{1}{x^2}$, $x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$, etc.

Examples: $10^{-1} = \frac{1}{10^1} = 0.1$, $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $36^{-1/2} = \frac{1}{\sqrt{36}} = \frac{1}{6}$, $9^{-5/2} = \frac{1}{\left(\sqrt{9}\right)^5} = \frac{1}{3^5} = \frac{1}{243}$.

Exponent Properties

$a^n a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$\left(a^n\right)^m = a^{nm}$

$a^0 = 1, \quad a \neq 0$

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$a^{-n} = \frac{1}{a^n}$

$\frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$

Properties of Radicals

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even