



# 3.1 Derivatives

Great Sand Dunes National Monument, Colorado

Photo by Vickie Kelly, 2003

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$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  is called the derivative of  $f$  at  $a$ .

We write:  $f'(x) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

“The derivative of  $f$  with respect to  $x$  is ...”

**There are many ways to write the derivative of  $y = f(x)$**



$f'(x)$  “f prime x” or “the derivative of f with respect to x”

$y'$  “y prime”

$\frac{dy}{dx}$  “dee why dee ecks” or “the derivative of y with respect to x”

$\frac{df}{dx}$  “dee eff dee ecks” or “the derivative of f with respect to x”

$\frac{d}{dx} f(x)$  “dee dee ecks uv eff uv ecks” or “the derivative of f of x”  
(  $d dx$  of  $f$  of  $x$  )



# Note:

$dx$  does not mean  $d$  times  $x$  !

$dy$  does not mean  $d$  times  $y$  !



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$\frac{dy}{dx}$  does not mean  $dy \div dx$  !

(except when it is convenient to think of it as division.)

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$\frac{d}{dx} f(x)$  does not mean  $\frac{d}{dx}$  times  $f(x)$  !

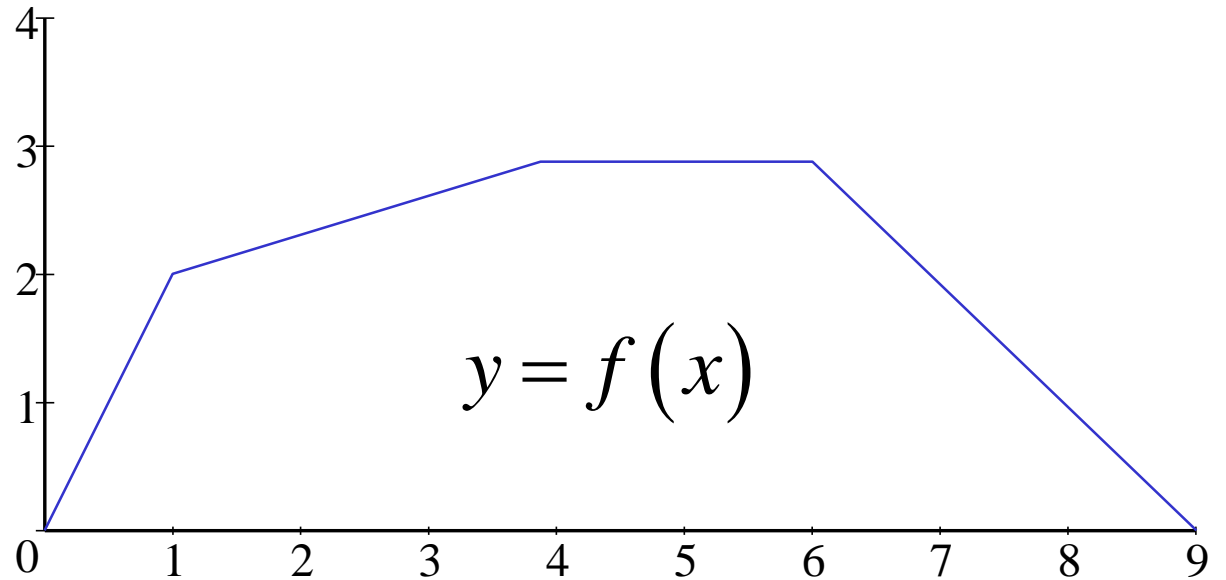
(except when it is convenient to treat it that way.)



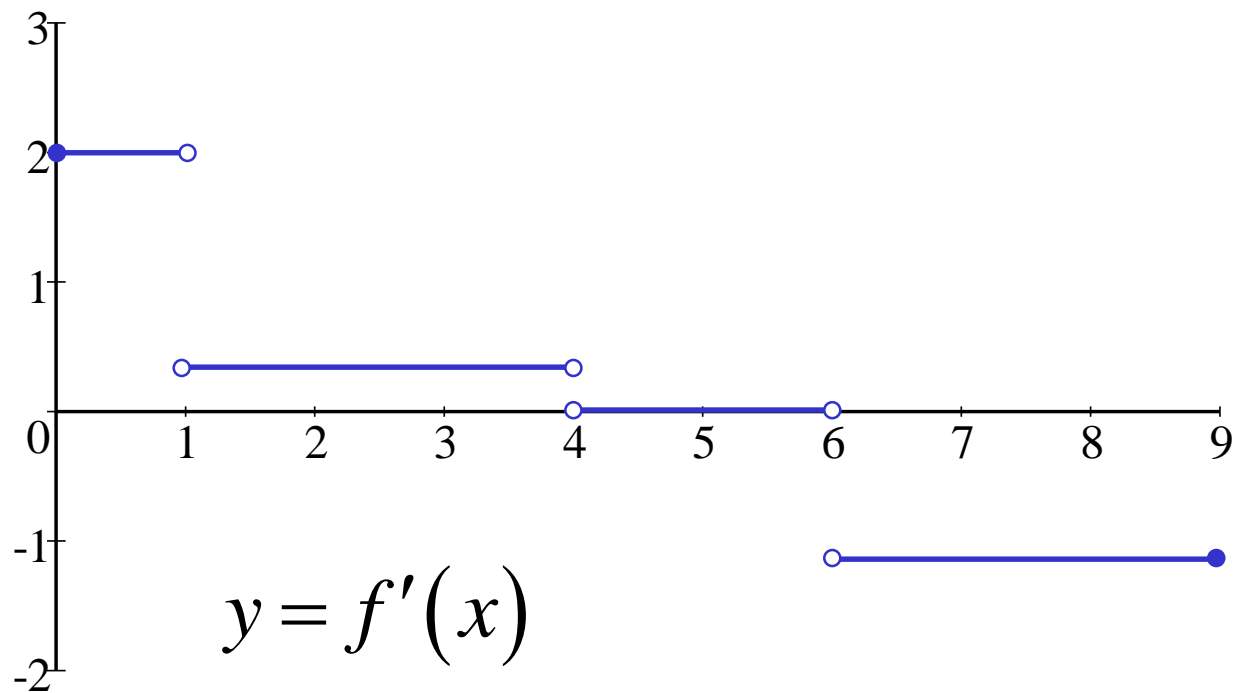




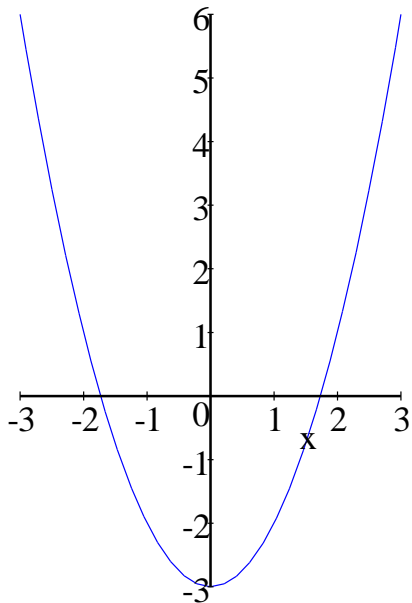
In the future, all will become clear. →



The derivative is the slope of the original function.

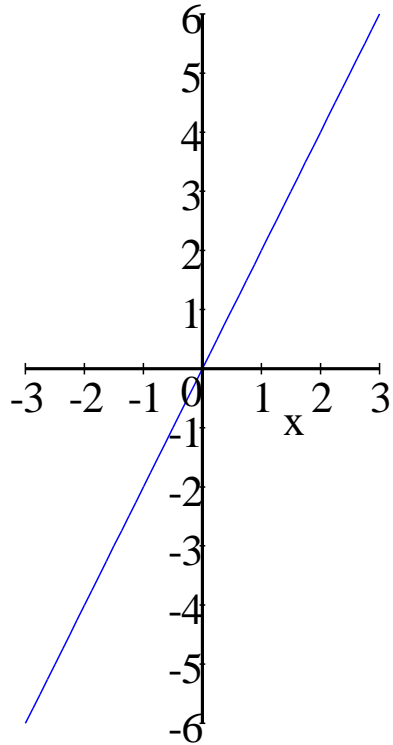






$$y = x^2 - 3$$

$$y' = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 3 - (x^2 - 3)}{h}$$



$$y' = \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2x\cancel{h} + \cancel{h^2} - \cancel{x^2}}{\cancel{h}}$$

$$y' = \lim_{h \rightarrow 0} 2x + \cancel{h} \overset{0}{\nearrow}$$

$$y' = 2x$$



A function is differentiable if it has a derivative everywhere in its domain. It must be continuous and smooth. Functions on closed intervals must have one-sided derivatives defined at the end points.

Assignment p.105 # 1-4, 13-16