3.2 Differentiability

Arches National Park

Photo by Vickie Kelly, 2003

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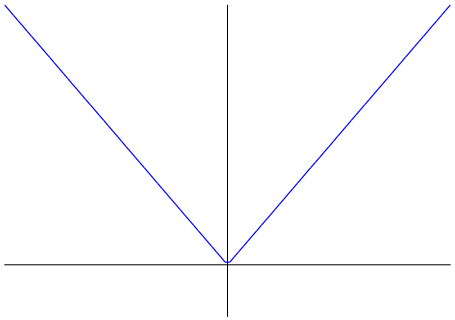
Greg Kelly, Hanford High School, Richland, Washington

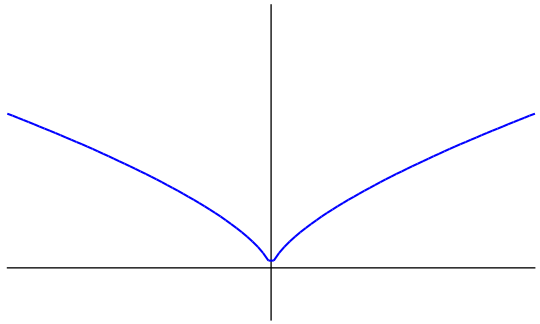
To be differentiable, a function must be continuous and smooth.

Derivatives will fail to exist at:

$f(x)=|x|$



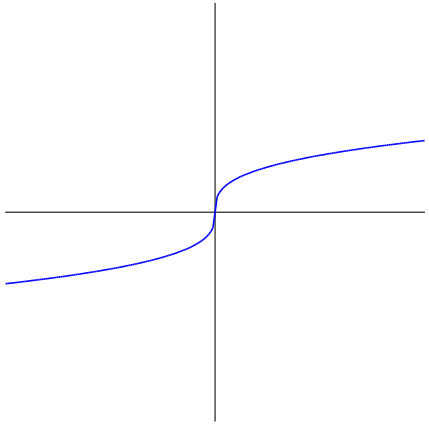
corner



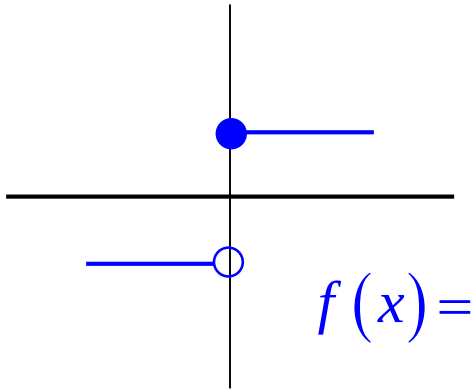
$f(x)=x^{\frac{2}{3}}$

cusp

$f(x)=\sqrt[3]{x}$



vertical tangent



$f(x)=\begin{cases}-1, & x<0\\ 1, & x\geq 0\end{cases}$

discontinuity

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Most of the functions we study in calculus will be differentiable.

Derivatives on the TI-84:

You must be able to calculate derivatives with the calculator and without.

Today you will be using your calculator, but be sure to do them by hand when called for.

Remember that half the test is no calculator.

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Example: $y = x^3$ Find $\frac{dy}{dx}$ at $x = 2$.

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Warning:

The calculator may return an incorrect value if you evaluate a derivative at a point where the function is not differentiable.

Examples: $d\left(1/x, x\right)\Big|\, x=0$ returns $-\infty$

$d\left(abs\left(x\right), x\right)\Big|\, x=0$ returns ± 1

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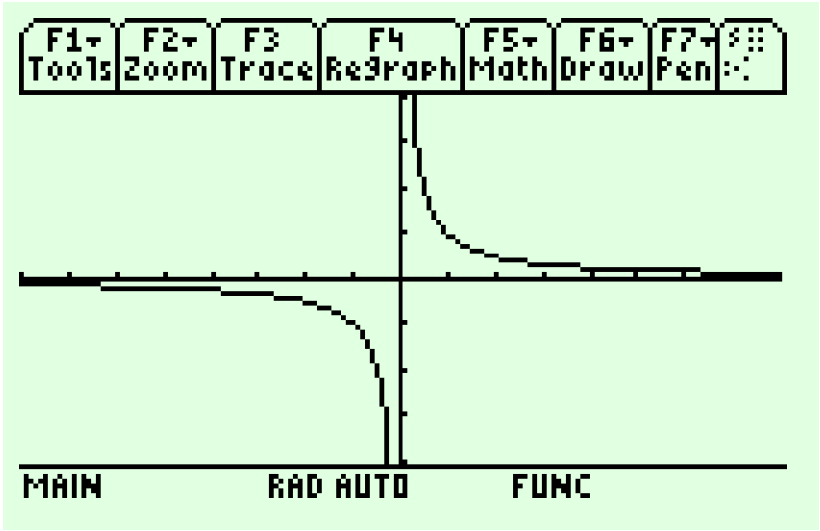
Graphing Derivatives

Graph: $y = d(\ln x, x)$ What does the graph look like?



This looks like: $y = \frac{1}{x}$

Use your calculator to evaluate: $d(\ln x, x)$

$\frac{1}{x}$

The derivative of $\ln x$ is only defined for $x > 0$, even though the calculator graphs negative values of x.

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There are two theorems on page 110:

If f has a derivative at $x = a$, then f is continuous at $x = a$.

Since a function must be continuous to have a derivative, if it has a derivative then it is continuous.

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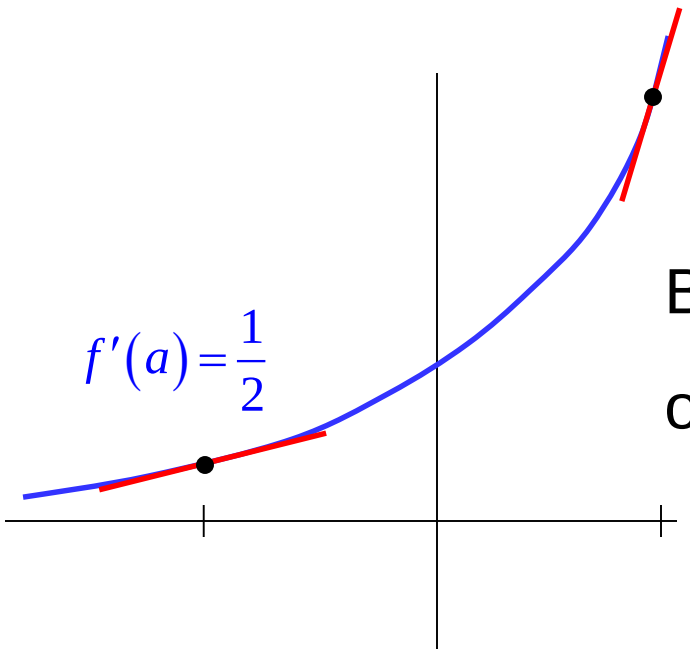
Intermediate Value Theorem for Derivatives

If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.



Between a and b, f' must take on every value between $\frac{1}{2}$ and 3.

π

Assignment p.105 # 1-4, 13-16

Assignment p.114 #5-10, 17-24