Warm-up

1. Is the function $y = |x+ 1|$ differentiable? Why?

2. Find dy/dx of $f(x) = x^2 + x - 1$ at $x = 1$.

3.3 Differentiation Rules

Colorado National Monument

Photo by Vickie Kelly, 2003

If the derivative of a function is its slope, then for a constant function, the derivative must be zero.

$$\boxed{\frac{d}{dx}(c)=0}$$

example: $y=3$

$$y'=0$$

The derivative of a constant is zero.

→

We saw that if $y = x^2$, $y' = 2x$.

This is part of a pattern.

$$\boxed{\frac{d}{dx}\left(x^n\right) = nx^{n-1}}$$

power rule

examples:

$f(x) = x^4$

$f'(x) = 4x^3$

$y = x^8$

$y' = 8x^7$

→

constant multiple rule:

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

examples:

$$\frac{d}{dx}cx^n = cnx^{n-1}$$

$$\frac{d}{dx}7x^5 = 7\cdot 5x^4 = 35x^4$$

$\rightarrow$

constant multiple rule:

$$\frac{d}{dx}(cu)=c\frac{du}{dx}$$

sum and difference rules:

$$\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$$

$$\frac{d}{dx}(u-v)=\frac{du}{dx}-\frac{dv}{dx}$$

$$y=x^4+12x$$

$$y'=4x^3+12$$

$$y=x^4-2x^2+2$$

$$\frac{dy}{dx}=4x^3-4x$$

→

Example:

Find the horizontal tangents of: $y = x^4 - 2x^2 + 2$

$$\frac{dy}{dx} = 4x^3 - 4x$$

Horizontal tangents occur when slope = zero.

$$4x^3 - 4x = 0$$

$$x^3 - x = 0$$

$$x\left(x^2 - 1\right) = 0$$

$$x\left(x+1\right)\left(x-1\right) = 0$$

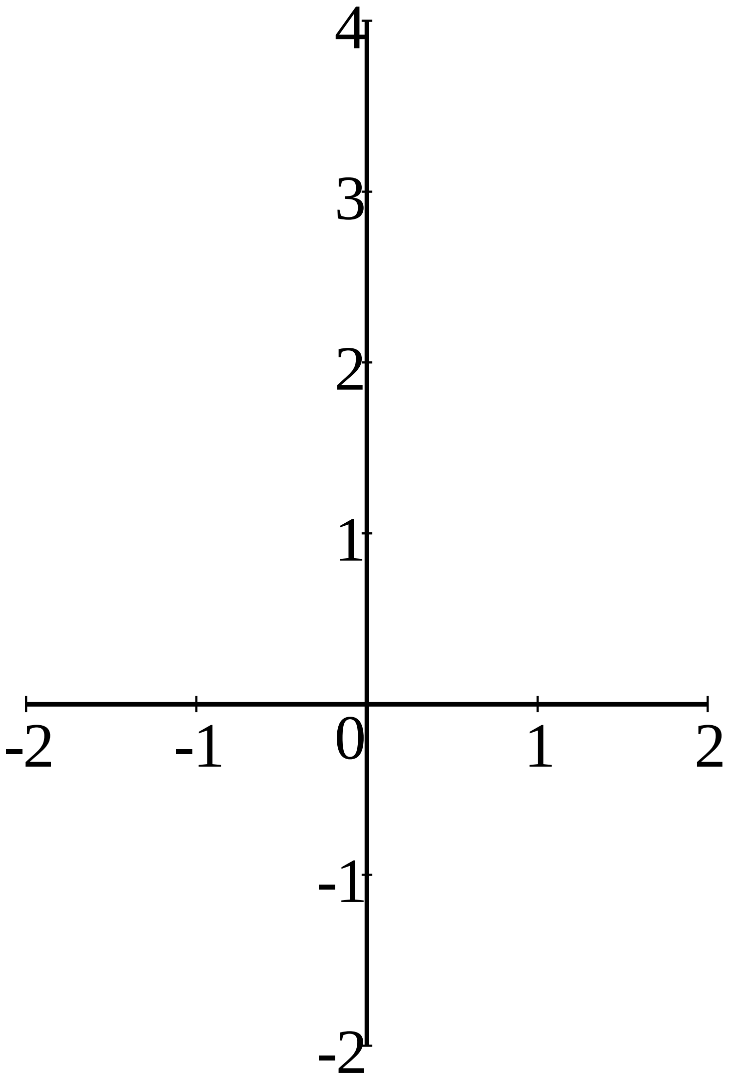
$$x = 0, -1, \ 1$$

Plugging the x values into the original equation, we get:

$$y = 2, \ y = 1, \ y = 1$$

(The function is even, so we only get two horizontal tangents.)

→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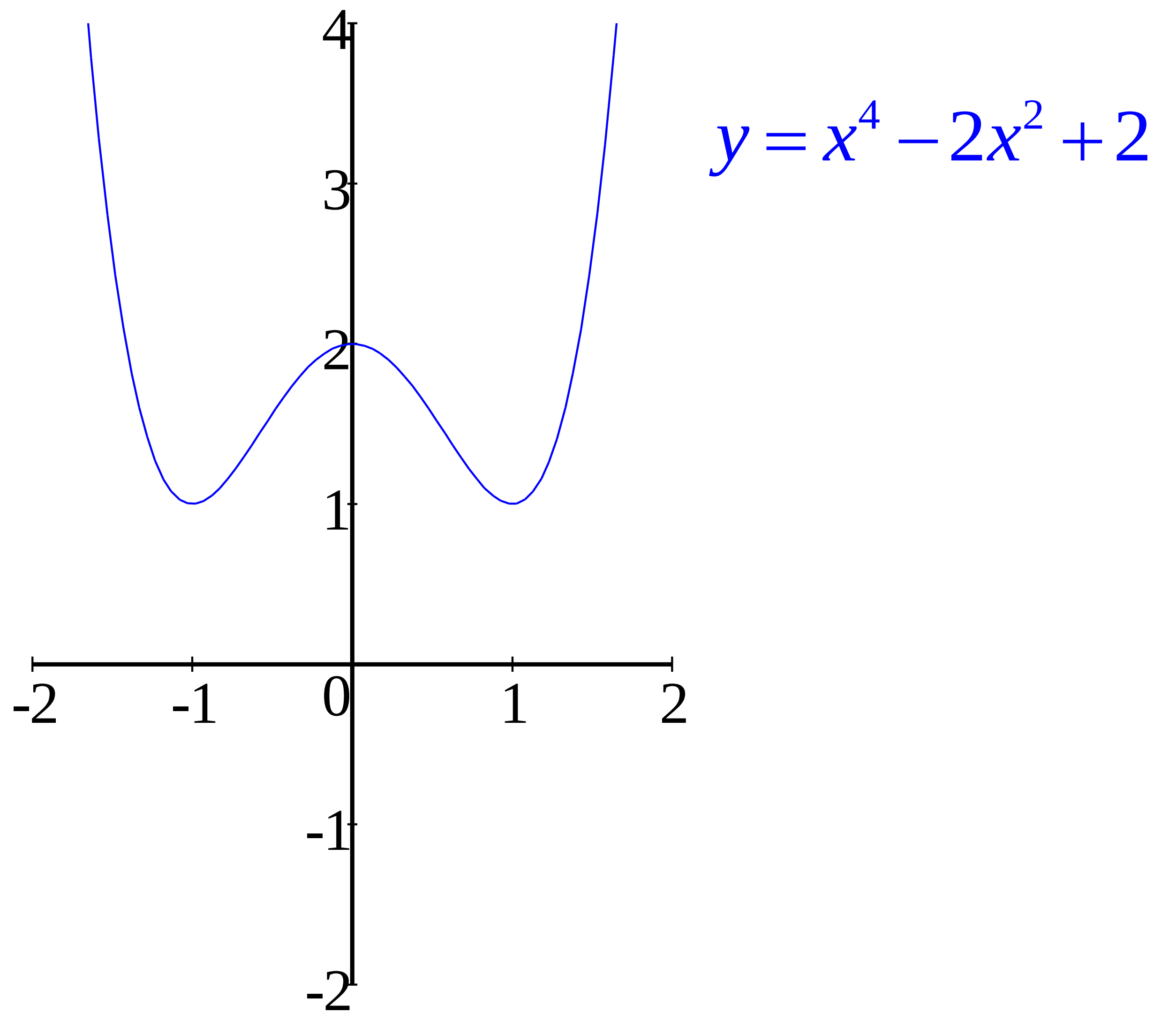
$y = x^4 - 2x^2 + 2$
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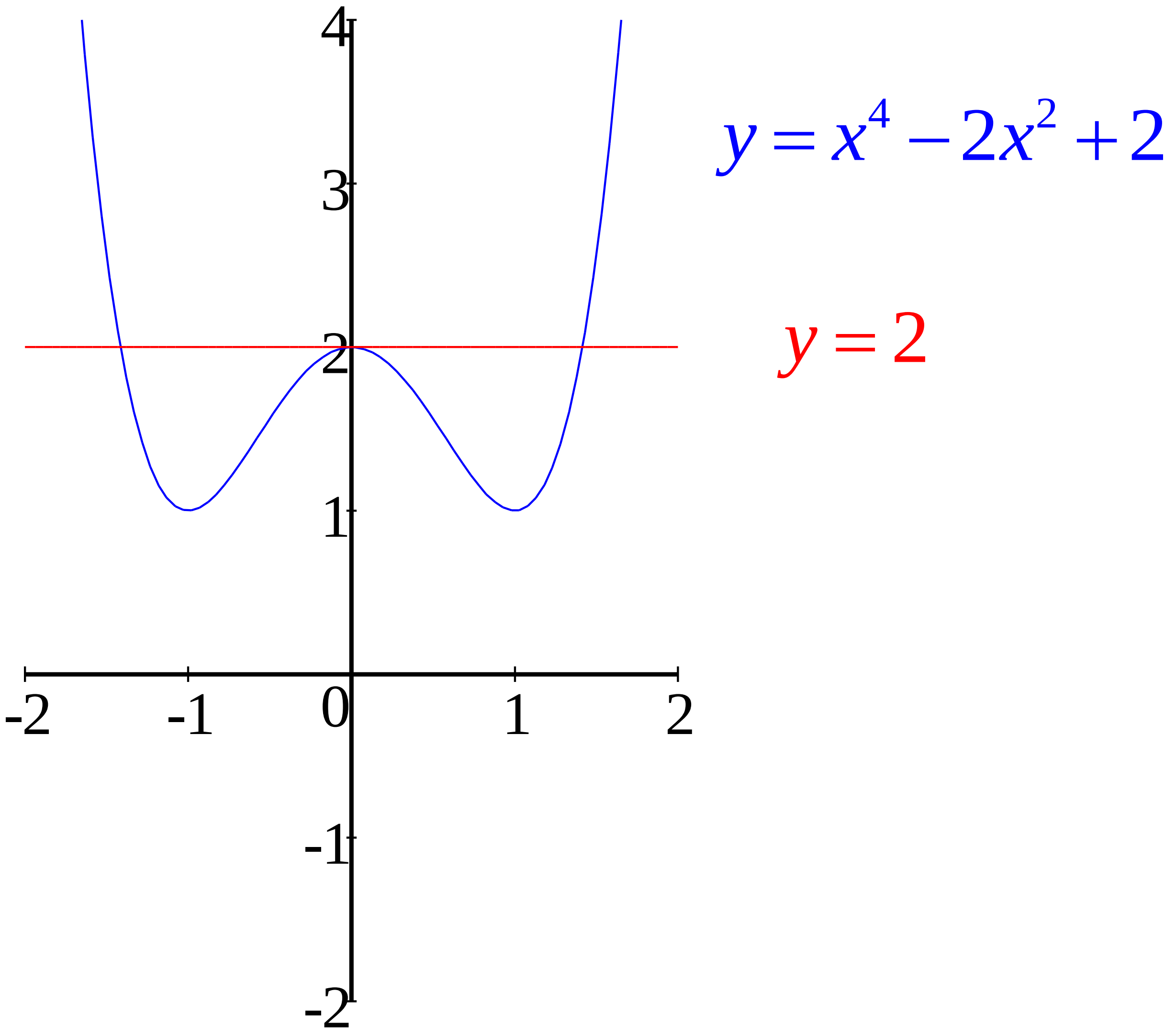
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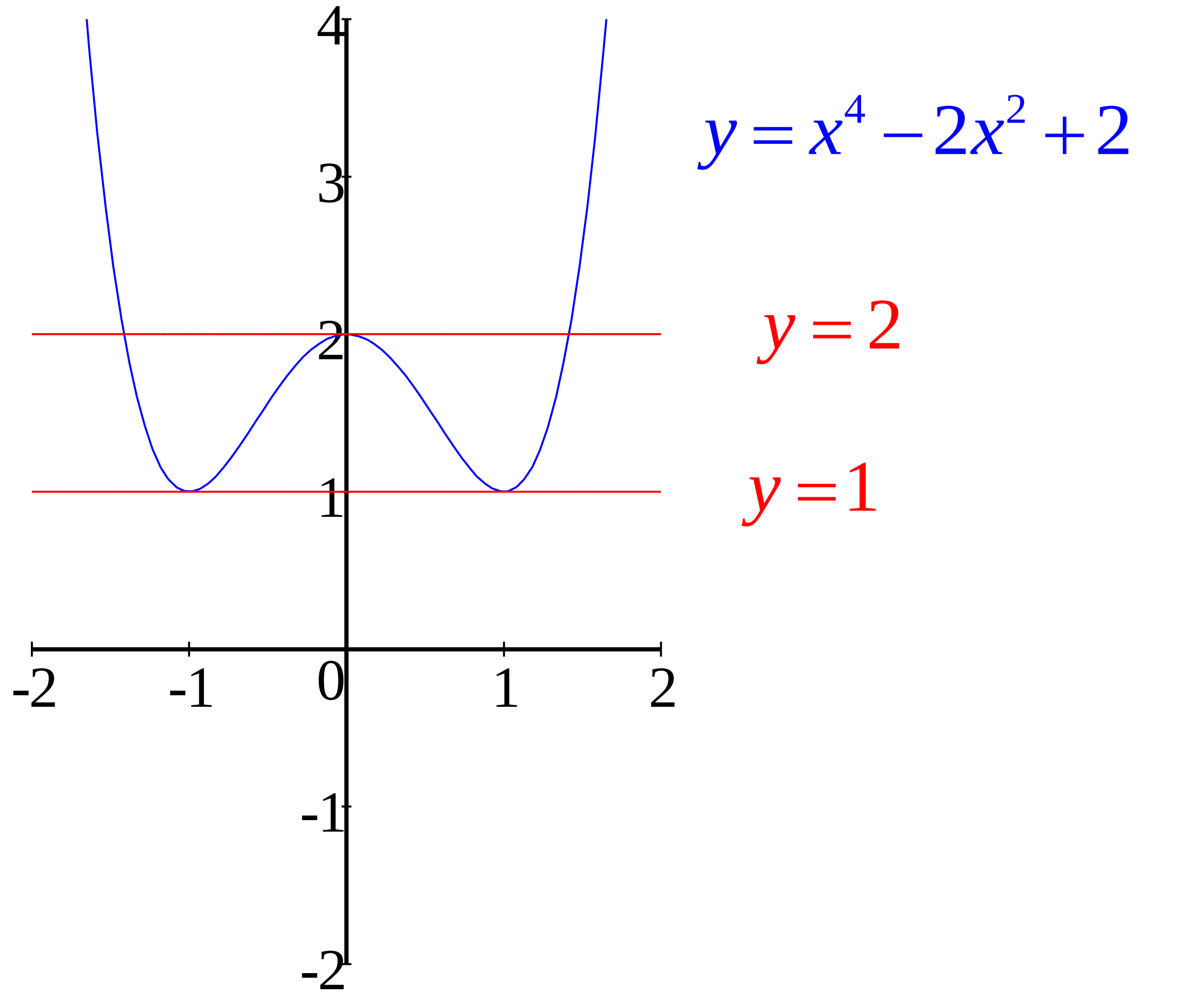
$y = x^4 - 2x^2 + 2$
$y = 2$



$y = x^4 - 2x^2 + 2$
$y = 2$
$y = 1$
4
3
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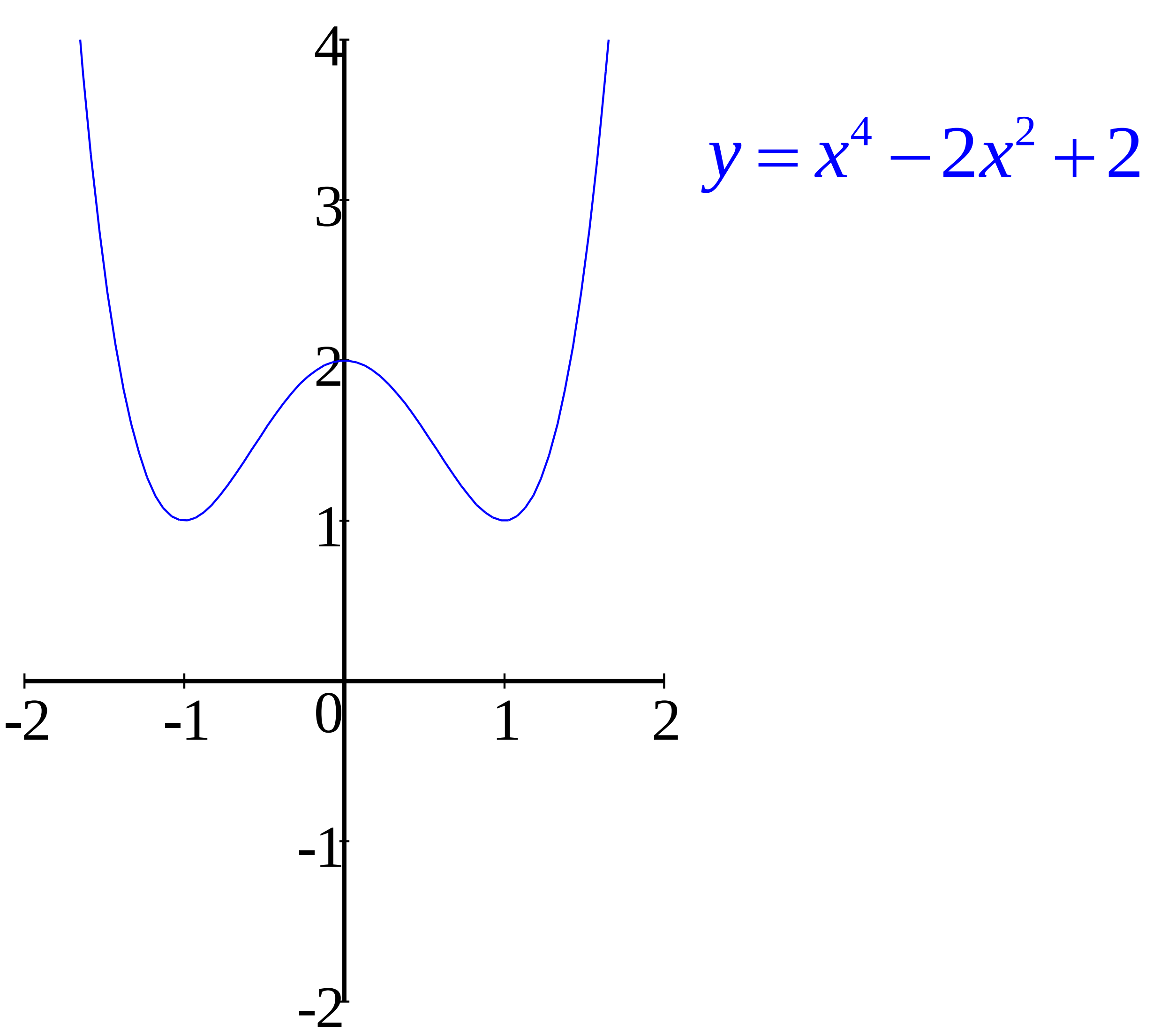
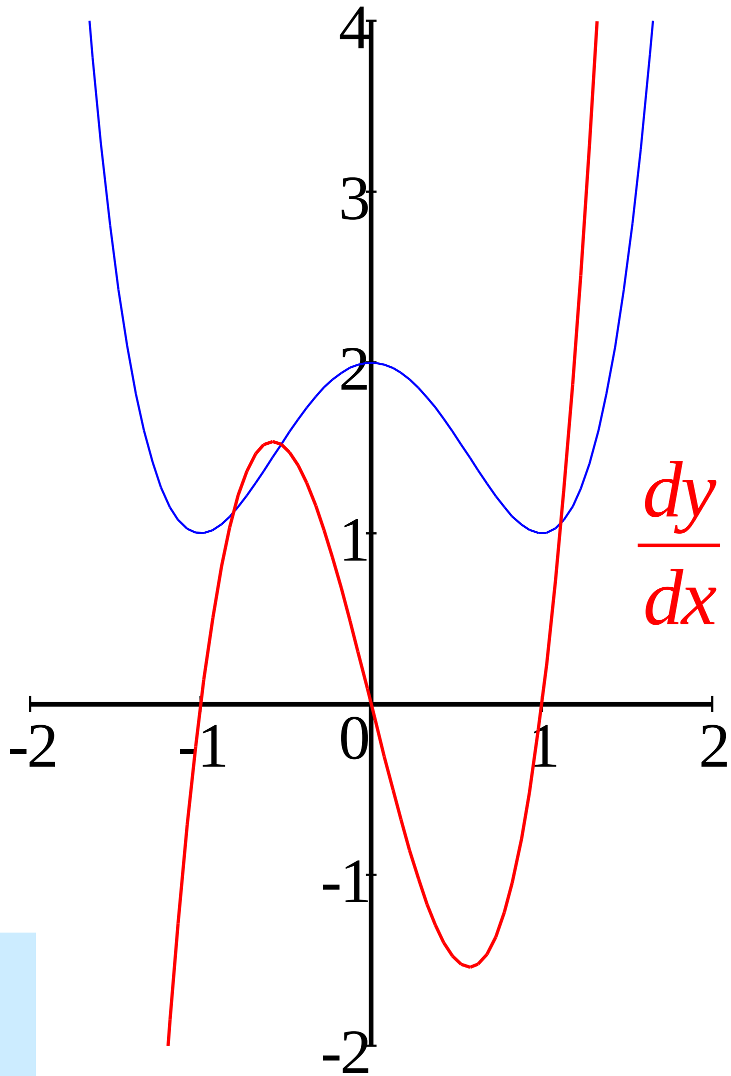
1
2



$y = x^4 - 2x^2 + 2$
4
3
2
1
-1
-2
-2
-1
0
1
2



$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

First derivative (slope) is zero at:

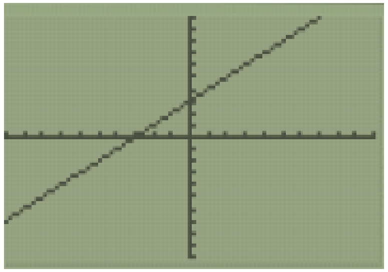
$$x = 0, -1, \ 1$$

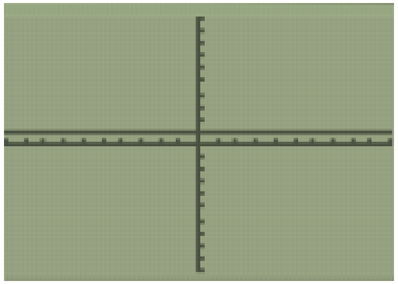
$\rightarrow$

Graph the original function then find its derivative and its graph.

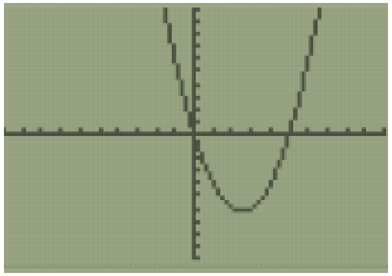
Original | Derivative

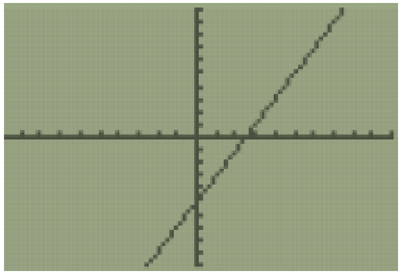
1. $f(x) = x + 3$



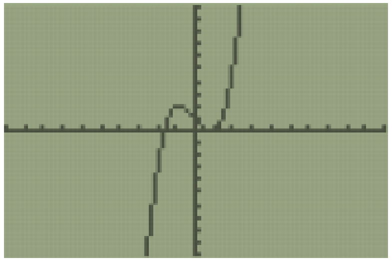


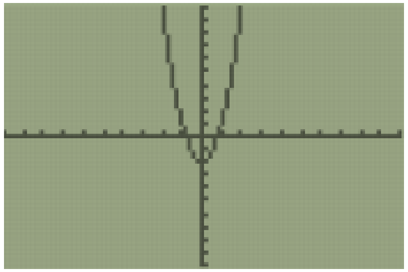
2. $f(x) = x^2 - 5x$





3. $f(x) = x^3 - 2x + 1$





Derivative Graph Activity

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product rule:

$$\boxed{\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}}$$

Notice that this is <u>not</u> just the product of two derivatives.

This is sometimes memorized as: $d(uv) = u\ dv + v\ du$

$$\frac{d}{dx}\left[\left(x^2+3\right)\left(2x^3+5x\right)\right] = \left(x^2+3\right)\left(6x^2+5\right) + \left(2x^3+5x\right)\left(2x\right)$$

$$\frac{d}{dx}\left(2x^5+5x^3+6x^3+15x\right)$$

$$\frac{d}{dx}\left(2x^5+11x^3+15x\right) \qquad 6x^4+5x^2+18x^2+15+4x^4+10x^2$$

$$10x^4+33x^2+15 \qquad 10x^4+33x^2+15$$

$\rightarrow$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$$

or

$$d\left(\frac{u}{v}\right)=\frac{v\,du-u\,dv}{v^2}$$

$$\frac{d}{dx}\frac{2x^3+5x}{x^2+3}=\frac{\left(x^2+3\right)\left(6x^2+5\right)-\left(2x^3+5x\right)\left(2x\right)}{\left(x^2+3\right)^2}$$

$\rightarrow$

Higher Order Derivatives:

$y' = \frac{dy}{dx}$ is the first derivative of y with respect to x.

$y'' = \frac{dy'}{dx} = \frac{d}{dx}\frac{dy}{dx} = \frac{d^2y}{dx^2}$ is the <u>second</u> derivative.

(y double prime)

$y''' = \frac{dy''}{dx}$ is the third derivative.

$y^{(4)} = \frac{d}{dx}y'''$ is the fourth derivative.

We will learn later what these higher order derivatives are used for.

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