Warm-up

1. If $f(x) = cos^3(4x)$ find $f'(x)$

2. If $f(x) = (2x - 3)^4$ find $f'(x)$

3. Find y' of y = log_5x

4. If $x^2 + 3xy^2 = 6$, what is $\frac{dy}{dx}$ at the point (-1, 3)?

3.4 Velocity, Speed, and Rates of Change

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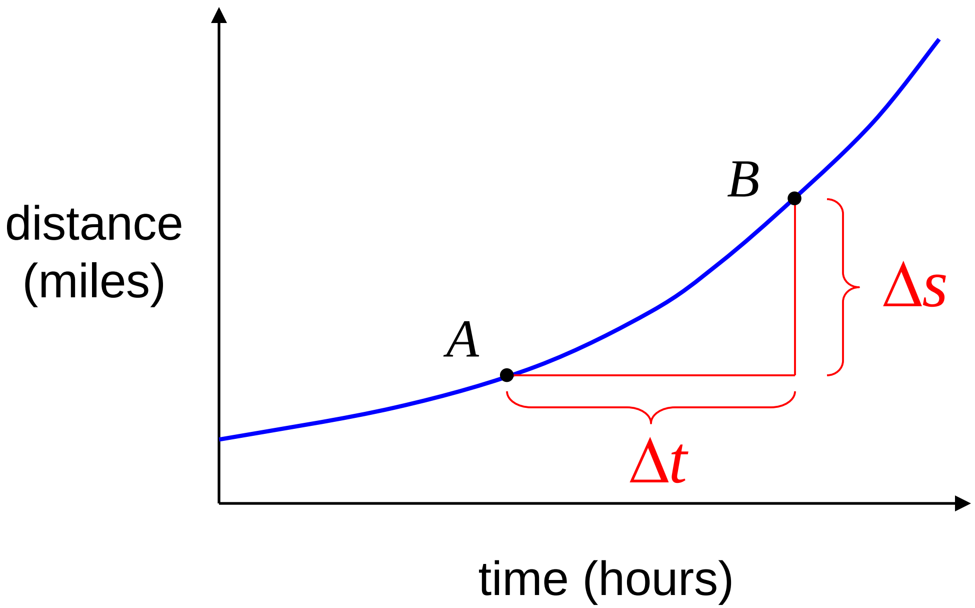
Consider a graph of displacement (distance traveled) vs. time.



Average velocity can be found by taking:

$$\frac{\text{change in position}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

$$V_{\text{ave}} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

The speedometer in your car does not measure average velocity, but <u>instantaneous velocity.</u>

(The velocity at one moment in time.)

$$V(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

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Velocity is the first derivative of position.

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Example: Free Fall Equation

$$s = \frac{1}{2} g\, t^2$$

$$s = \frac{1}{2} \cdot 32\, t^2$$

$$s = 16\, t^2$$

$$V = \frac{ds}{dt} = 32\, t$$

Gravitational Constants:

$$g = 32 \ \frac{\text{ft}}{\text{sec}^2}$$

$$g = 9.8 \ \frac{\text{m}}{\text{sec}^2}$$

$$g = 980 \ \frac{\text{cm}}{\text{sec}^2}$$

Speed is the absolute value of velocity.

→

Acceleration is the derivative of velocity.

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

example: $v = 32t$

$a = 32$

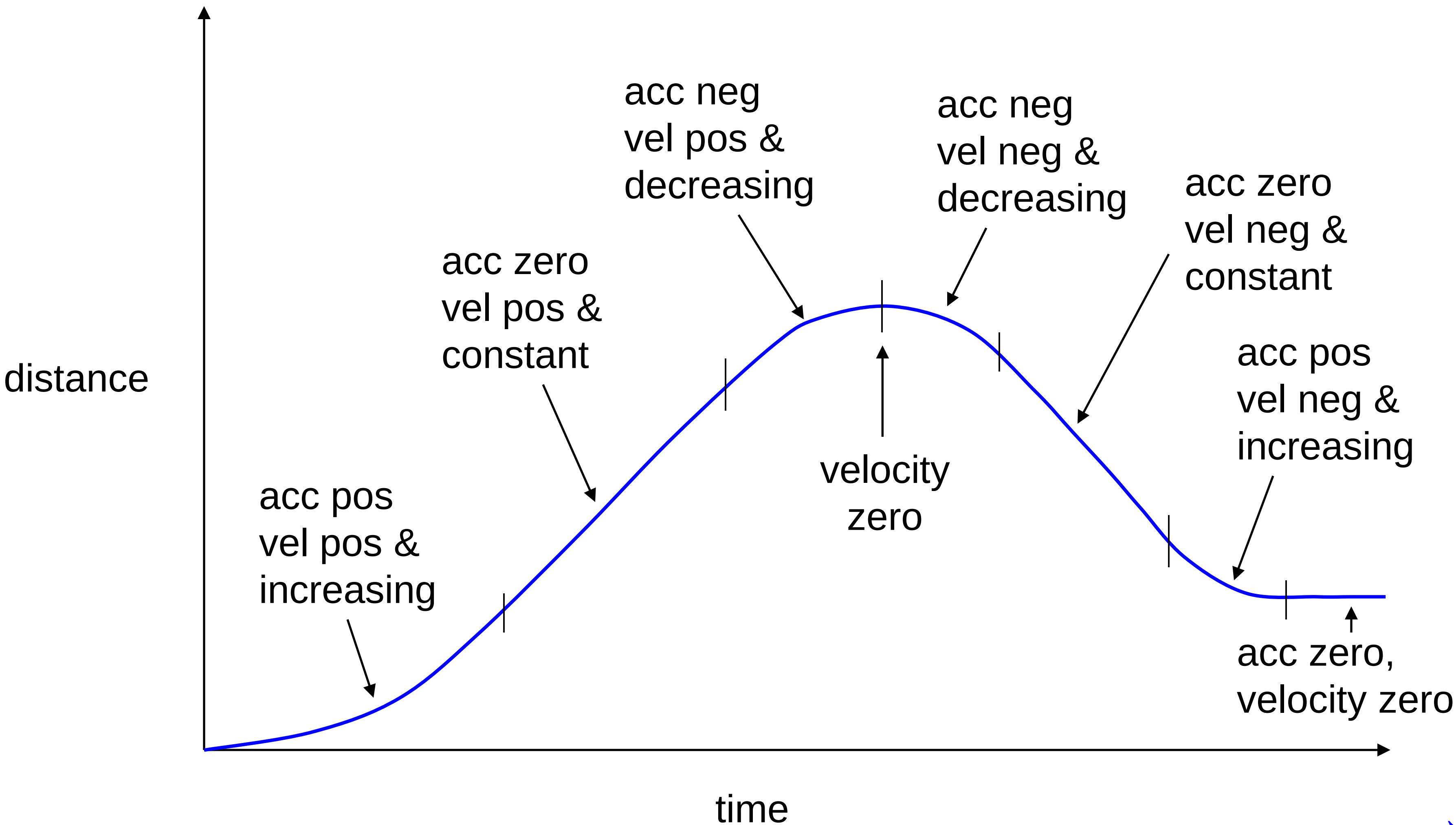
If distance is in: feet

Velocity would be in: $\frac{\text{feet}}{\text{sec}}$

Acceleration would be in: $\frac{\frac{\text{ft}}{\text{sec}}}{\text{sec}} = \frac{\text{ft}}{\text{sec}^2}$

$\rightarrow$

It is important to understand the relationship between a position graph, velocity and acceleration:



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Ex. A particle moves along the curve given by $y = \sqrt{t^3 + 1}$. Find the acceleration when t=2 seconds.

Ex. Suppose that position equation for a moving object is given by $s(t) = 3t^2 - 2t + 5$ where s is measured in meters and t is measured in seconds. Find the velocity of the object when t=2.

Rates of Change:

<u>Average</u> rate of change = $\dfrac{f(x+h)-f(x)}{h}$

<u>Instantaneous</u> rate of change = $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

These definitions are true for any function.

(x does not have to represent time.)

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p. 136 #8-10, 13, 20 - 24

Example 1:

For a circle:

$$A = \pi r^2$$

$$\frac{dA}{dr} = \frac{d}{dr} \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

Instantaneous rate of change of the area with respect to the radius.

$$dA = 2\pi r \; dr$$

For tree ring growth, if the change in area is constant then dr must get smaller as r gets larger.

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from Economics:

Marginal cost is the first derivative of the cost function, and represents an approximation of the cost of producing one more unit.

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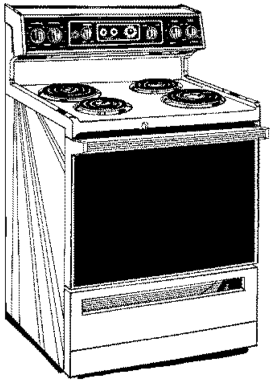
Example 13:



Suppose it costs: $c\left(x\right)=x^3-6x^2+15x$

to produce x stoves. $c'\left(x\right)=3x^2-12x+15$

If you are currently producing 10 stoves, the 11th stove will cost approximately:

$$c'\left(10\right)=3\cdot 10^2-12\cdot 10+15$$

$$=300-120+15$$

$$=\$195$$

marginal cost

Note that this is not a great approximation – Don’t let that bother you.

The actual cost is: $C\left(11\right)-C\left(10\right)$

$$=\left(11^3-6\cdot 11^2+15\cdot 11\right)-\left(10^3-6\cdot 10^2+15\cdot 10\right)$$

$$=770-550 \quad =\$220$$

actual cost

→

Note that this is not a great approximation – Don’t let that bother you.

Marginal cost is a linear approximation of a curved function. For large values it gives a good approximation of the cost of producing the next item.

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