

**Goal 1: The Definition of the Derivative**

- 1) What does the word “derivative” mean? What does it represent? What are the different types of notation used to represent the derivative?
- 2) What is the formula that is used to calculate the derivative of functions?
- 3) What is the difference between finding the average rate of change and finding the derivative?
- 4) An object is dropped from the top of a 100-m tower. Its height above the ground after  $t$  seconds is  $h=100 - 4.9t^2$ .
  - a. Find the object’s **average rate** of change on the interval  $[1, 3]$
  - b. Find the object’s instantaneous rate of change at  $t = 2$
- 5) Given:  $f(x) = 3x^2 + 6$  Find  $f'(x)$  using the “limit” definition. Check your answer using the power rule.

**Goal 2: Differentiability**

- 1) What does “differentiable” mean?
- 2) For a function to be differentiable at a value what must happen at that point?
- 3) Describe the different situations in which the derivative will not exist at a particular point
- 4) If the derivative exists at the point where  $x = 4$ , MUST the function be continuous at  $x = 4$ ? Explain.
- 5) If a function is continuous at the point where  $x = 4$ , MUST the derivative exist at  $x = 4$ ? Explain.
- 6) Is the function  $y = \sqrt{|x|}$  continuous at  $x = 0$ ? Is it differentiable? Explain.
- 7) Determine where the function  $f(x) = \sqrt[5]{x-4}$  is not differentiable and explain why.
- 8) Show (using Calculus!) that the function  $f(x) = \begin{cases} \sqrt{x} & x \leq 1 \\ 2x - 1 & x > 1 \end{cases}$  is not differentiable at  $x = 1$

### Goal 3: Derivative Rules

Differentiate each of the following.

1.  $y = x \sin(x)$

2.  $f(x) = e^{6x}$

3.  $y = \frac{1-x^2}{1+x^2}$

4.  $y = (x^3 - 1)^5$

5.  $y = \tan(2x)$

6.  $y = \tan x \sec x$

7.  $y = \sqrt{\ln x}$

8.  $y = \frac{x^4 - 1}{x^2}$

9. Find  $\frac{d^2y}{dx^2}$  of  $y = \frac{1}{x^2}$

10.  $y = \sqrt[3]{2x^2 + 4x + 1}$

11.  $y = x^5 \sec x$

12.  $y = \cos(3x)$

13.  $y = 2x \cos(x)$

14.  $y = \sin(3x)\sin(x^2)$

15. Use the information below to find  $h'(2)$  if  $h(x) = f(x)g(x)$

$$f(2) = 3, f'(2) = 4, g(2) = 3, g'(3) = 2$$

16. Find the equation for the tangent line to the graph of  $f(x) = \sqrt{x+1}$  at the point where  $x=3$ .

### Goal 4: Implicit Differentiation

Use implicit differentiation to find  $\frac{dy}{dx}$

1.  $x^2 + 2y^2 = 2xy$

2.  $y^3 + xy^2 = \frac{3}{2}$

3.  $3y^3 - 4x^2y + xy = -5$

4. Find the equation of the normal line of  $x^2 + xy - y^2 = 1$  at  $(1, 0)$