Warm-up

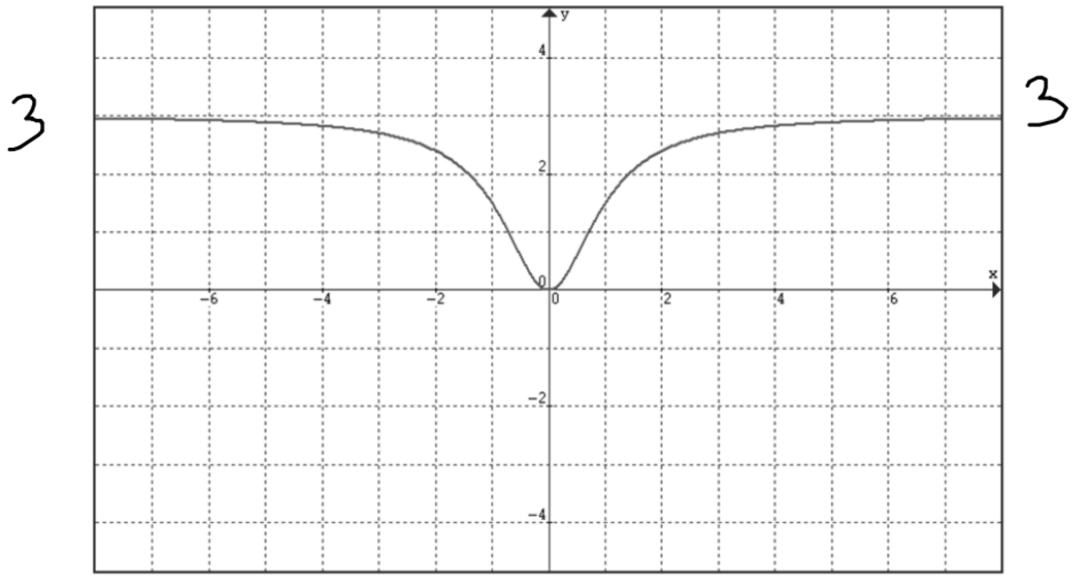
$$\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$$

Unit 1 Day 3:
Limits at Infinity

$$\infty$$

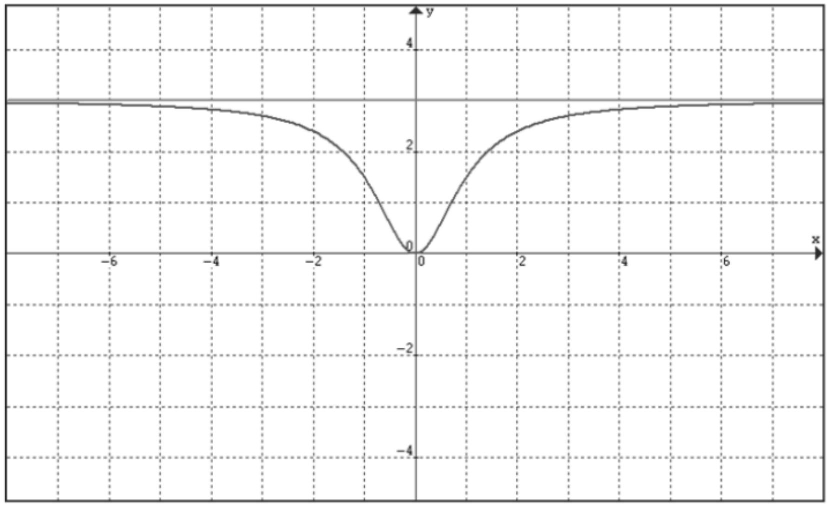
$$f(x) = \frac{3x^2}{x^2+1}$$

- Discuss "end behavior" of a function on an interval
- Graph:



$$\lim_{x \to \infty} \frac{3x^2}{x^2 + 1}$$

- As x increases without bound f(x) approaches ________.
- As x decreases without bound f(x) approaches ________.



Definition of a Horizontal Asymptote

- The line y=L is a ***horizontal asymptote*** of the graph of *f* if

$$\lim_{x \to -\infty} f(x) = L$$

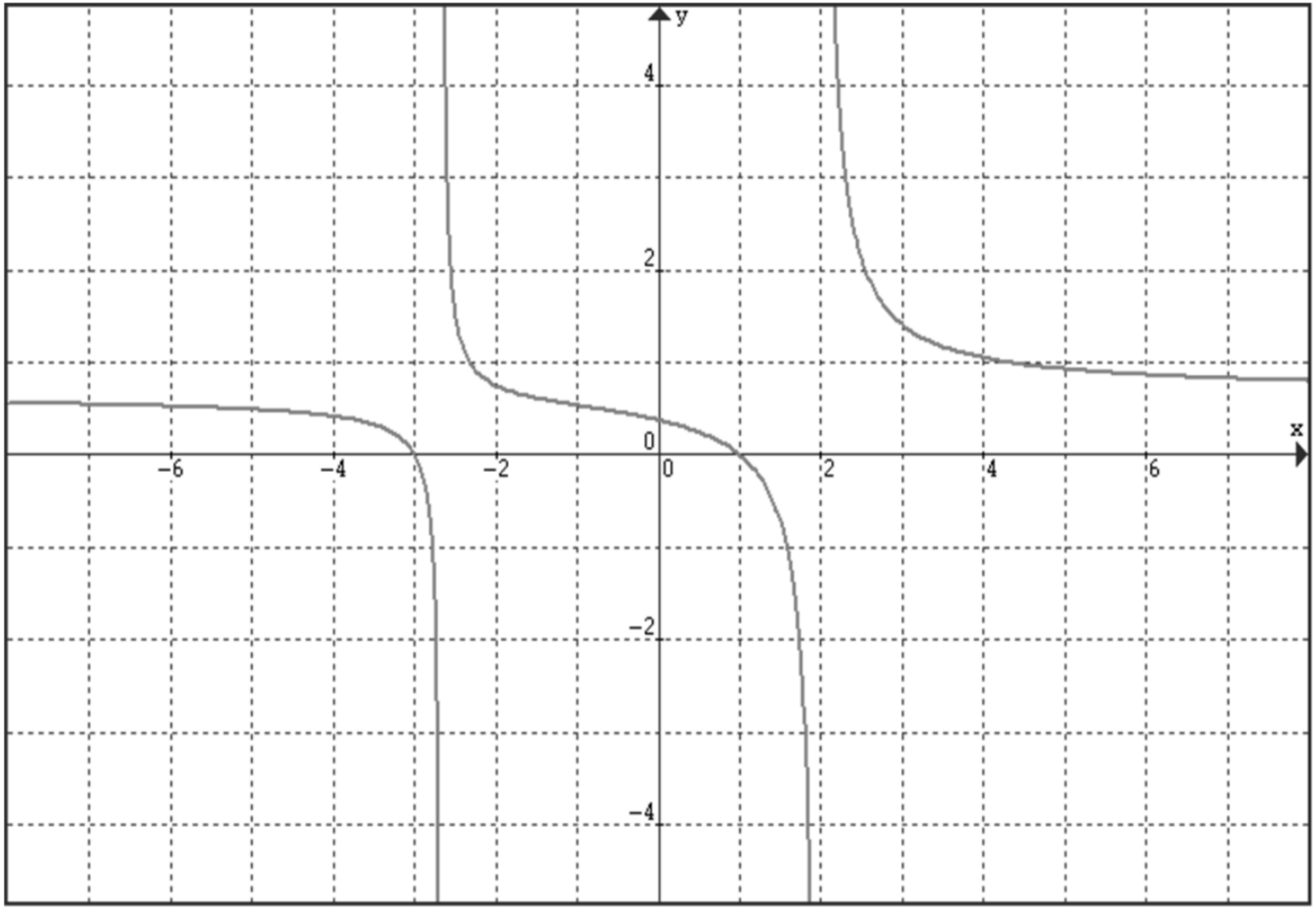
OR

$$\lim_{x \to +\infty} f(x) = L$$

Note: a function can have at most 2 horizontal asymptotes

Exploration

- Use a graphing utility to graph
  $y=(2x^2 +4x-6)/(3x^2+2x-16)$
- Describe all important features of the graph.
- Can you find a single viewing window that shows all these features clearly?
- What are the horizontal asymptotes?


y
x
4
2
0
−2
−4
−6
−4
−2
0
2
4
6

Rules for Limits at infinity:

1. If r is a positive rational number and c is any real number, then $\lim_{x\to\infty} \frac{c}{x^r} = 0$

   Furthermore, if x^r is defined when x<0, then $\lim_{x\to-\infty} \frac{c}{x^r} = 0$

2. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.

3. If the degree of the numerator is *equal* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.

4. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function DNE, but we say ∞ or -∞.

Examples

1) $$\lim_{x\to\infty}(4-\frac{2}{x^2})$$

2) $$\lim_{x\to\infty}(\frac{2x-3}{x+1})$$

3) $$\lim_{x\to\infty}(\frac{2x+5}{3x^2+1})$$

4) $$\lim_{x\to\infty}(\frac{2x^3+5}{3x^2+1})$$

You try!

1) $$\lim_{x\to\infty} x^3$$

2) $$\lim_{x\to-\infty} x^3$$

3) $$\lim_{x\to\infty} \frac{2x^2 - 4x}{x+1}$$

4) $$\lim_{x\to-\infty} \frac{2x^2 - 4x}{x+1}$$

DEFINITION Vertical Asymptote

The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if either

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$

Recall finding vertical asymptotes:
When given a function, find a value for x which makes the function undefined. The value of x will be a vertical asymptote.

Given

$$f(x) = \frac{x^2 - x - 6}{x^2 - 4} = \frac{(x-3)(x+2)}{(x-2)(x+2)}$$

we see that the function **f** is discontinuous at **x** = -**2** and **x** = **2**. Since

$$\lim_{x\to -2} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x\to -2} \frac{x-3}{x-2} = \frac{5}{4}$$

x = -**2** is a removable discontinuity.

Since

$$\lim_{x \to 2+} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x \to 2+} \frac{x - 3}{x - 2} = -\infty$$

and

$$\lim_{x \to 2-} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x \to 2-} \frac{x - 3}{x - 2} = +\infty$$

x = 2 is a vertical asymptote for **f**.

$$f(x) = \frac{2x + 4}{x^2 - 4}$$